DATE: _______________

Extreme Values of Functions

What is an Extrema?

Extremum – point where the maximum (largest y-value) or minimum (smallest y-value) occurs

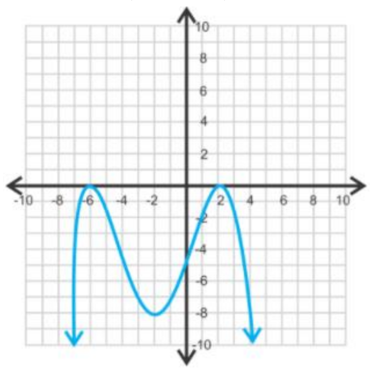
Extrema – points where the maxima (largest y-value) or minima (smallest y-value) occur

Extreme Value – y-value of the maxima or minima

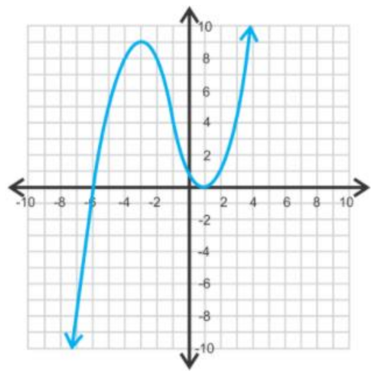
✱Absolute Maximum Value – THE highest y-value on entire graph or the given interval. (Global)

symbolically → $f(c) \geq f(x) \ \forall x$

graphically → absolute maximum value on $(-\infty, \infty)$



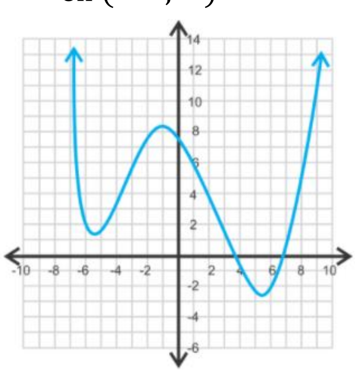
absolute maximum value on $(-\infty, 3]$



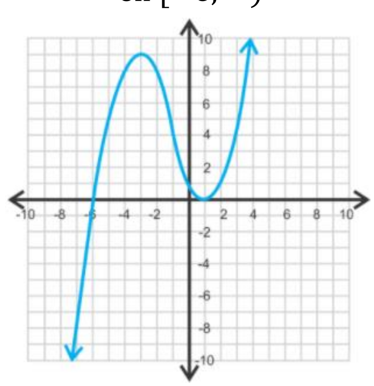
✱Absolute Minimum Value – THE lowest y-value on entire graph or the given interval. (Global)

symbolically → $f(c) \leq f(x) \ \forall x$

graphically → absolute minimum value on $(-\infty, \infty)$



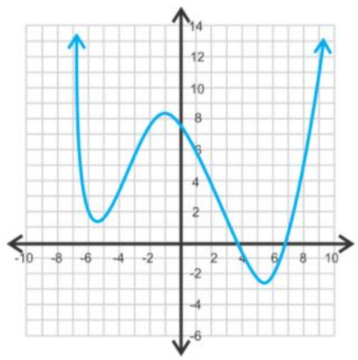
absolute minimum value on $[-6, \infty)$



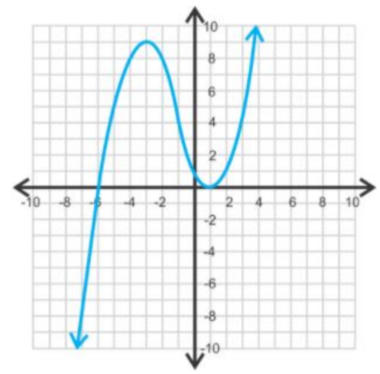
✱Relative Maximum Value – highest y-value on an open interval. (think: where $f(x)$ changes from increasing to decreasing)
(Local)

symbolically → $f(c) \geq f(x)$ $\forall x$ on (a, b)

graphically → relative maximum value on $(-\infty, \infty)$



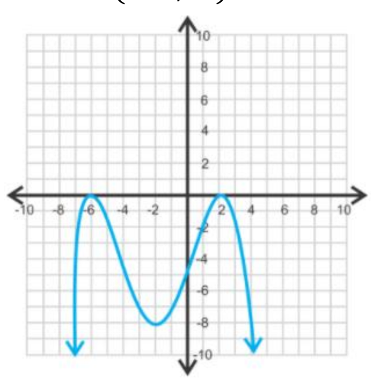
relative maximum value on $(-\infty, \infty)$



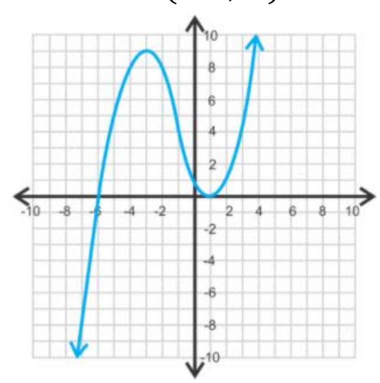
✱Relative Minimum Value – lowest y-value on an open interval. (think: where $f(x)$ changes from decreasing to increasing)
(Local)

symbolically → $f(c) \leq f(x)$ $\forall x$ on (a, b)

graphically → relative minimum value on $(-\infty, \infty)$



relative minimum value on $(-\infty, \infty)$



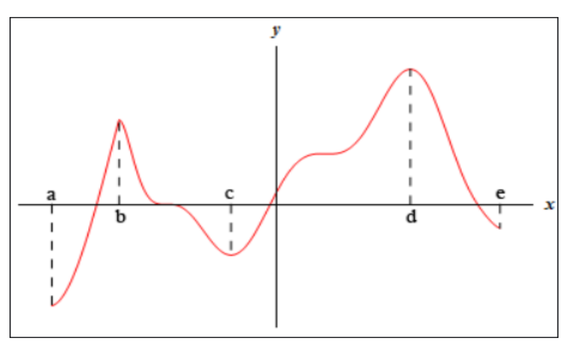
Example:
Which indicated x-value in the drawing to the right has:

a relative minimum value?

a relative maximum value?

an absolute minimum value?

an absolute maximum value?

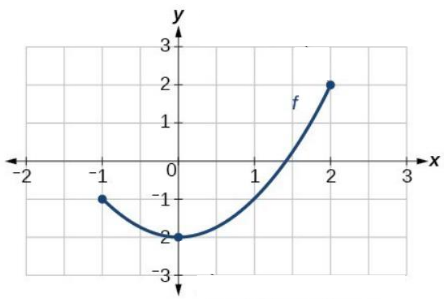


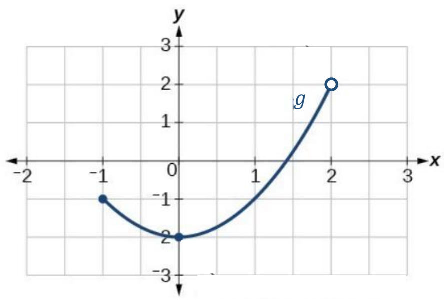
Extreme Value Theorem (EVT)

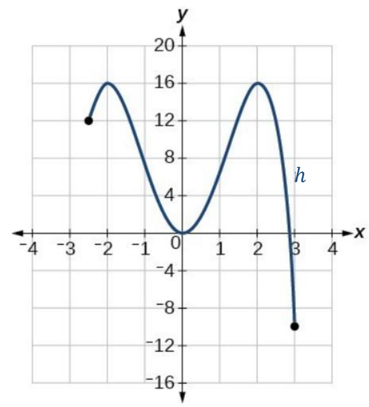
Extreme Value Theorem

If f is continuous on $[a, b]$,
then f has both an absolute maximum and absolute minimum on that interval.
(f has extreme values)

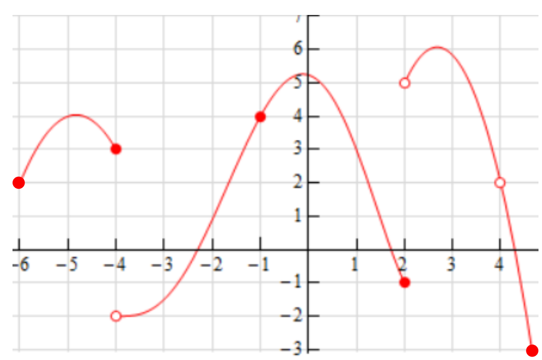
Examples:







EVT guarantees that an absolute maximum or minimum occurs,
but absolute maxima and absolute minima COULD exist
even if EVT doesn't guarantee their existence.



Graph of $f(x)$

The Extreme Value Theorem ONLY tells us that we CAN find an extreme value if a function is continuous.