

What is an Extrema?

Extremum – point where the maximum (largest y -value) or minimum (smallest y -value) occurs

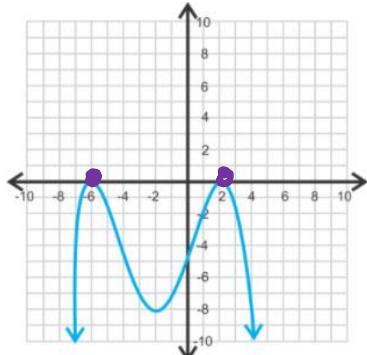
Extrema – points where the maxima (largest y -value) or minima (smallest y -value) occur

Extreme Value – y -value of the maximum or mimima

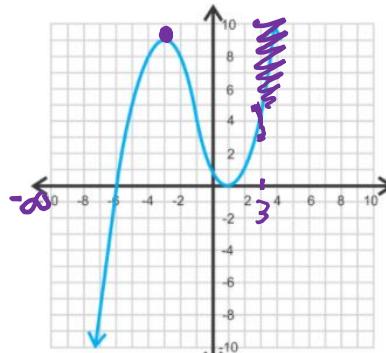
***Absolute Maximum Value** – THE highest y -value on entire graph or the given interval.
(Global)

symbolically $\rightarrow f(c) \geq f(x) \quad \forall x$
for all
some y-value | all other y-values
greater than or equal to

graphically \rightarrow absolute maximum value
on $(-\infty, \infty)$



absolute maximum value
on $(-\infty, 3]$

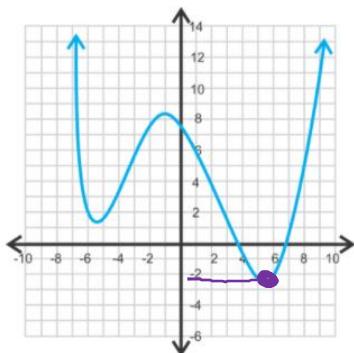


***Absolute Minimum Value** – THE lowest y -value on entire graph or the given interval.

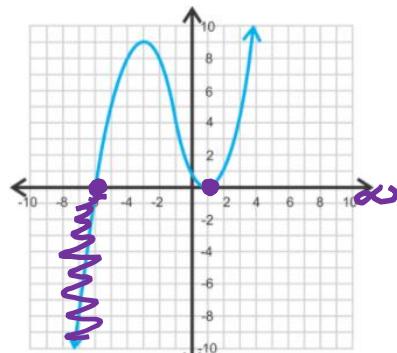
(Global)

symbolically $\rightarrow f(c) \leq f(x) \quad \forall x$
for all
some y-value | all other y-values
less than or equal to

graphically \rightarrow absolute minimum value
on $(-\infty, \infty)$



absolute minimum value
on $[-6, \infty)$



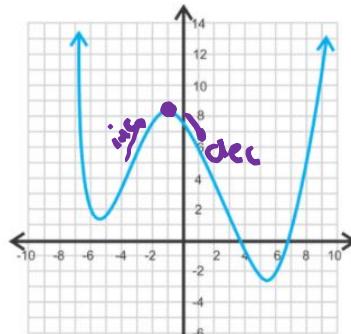
*Relative Maximum Value – highest y -value on an open interval. (think: where $f(x)$ changes from increasing to decreasing)
(Local)

symbolically $\rightarrow f(c) \geq f(x) \quad \forall x \text{ on } (a, b)$

some y-values greater than or equal to all other y-values on (a, b)

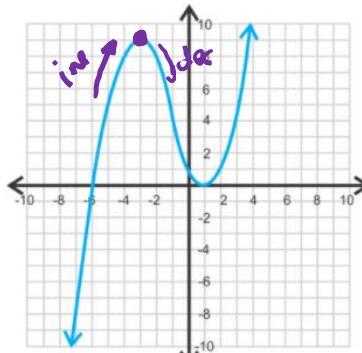
↑ rel. max

graphically \rightarrow relative maximum value on $(-\infty, \infty)$



rel. max value is 8.5

relative maximum value on $(-\infty, \infty)$



rel. max value is 9.

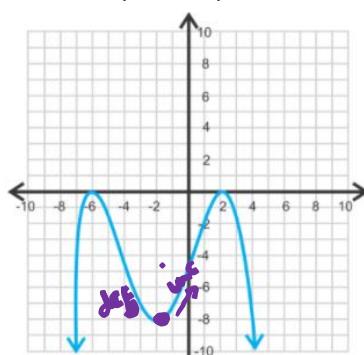
*Relative Minimum Value – lowest y -value on an open interval. (think: where $f(x)$ changes from decreasing to increasing)
(Local)

symbolically $\rightarrow f(c) \leq f(x) \quad \forall x \text{ on } (a, b)$

some y-value less than or equal to all other y-values on (a, b)

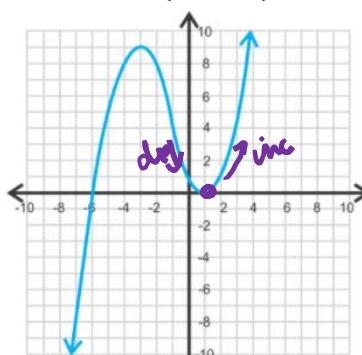
↑ rel. min

graphically \rightarrow relative minimum value on $(-\infty, \infty)$



rel. min value of -8

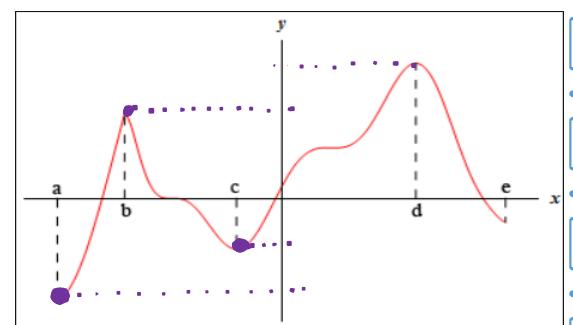
relative minimum value on $(-\infty, \infty)$



rel. min value of 0

Example:

- Which indicated x -value in the drawing to the right has:
 - a relative minimum value? **c**
 - a relative maximum value? **b**
 - an absolute minimum value? **a**
 - an absolute maximum value? **d**



Extreme Value Theorem (EVT)

Extreme Value Theorem

no jump holes in 

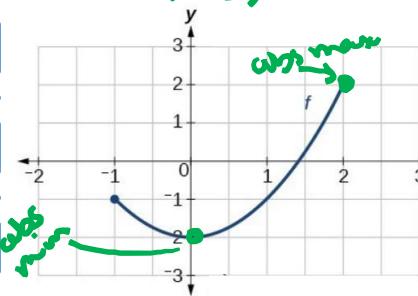
If f is continuous on $[a, b]$,

then f has both an absolute maximum and absolute minimum on that interval.

(f has extreme values)

Examples:

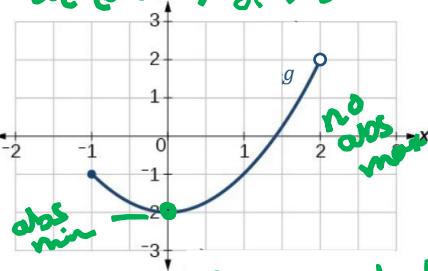
on $[-1, 2]$, f is cont.



\therefore , by EVT, f has an abs. max and abs min values.

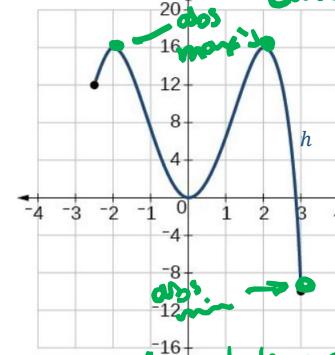
on $[-1, 2]$, g is not cont.

b/c @ $x = \sqrt{2}$, $g(2)$ DNE



$\therefore g$ is not guaranteed to have an abs max or abs min value.

on $[-2.5, 3]$, h is cont.



\therefore , by EVT h has an abs max and abs min value.

EVT guarantees that an absolute maximum or minimum occurs, but absolute maxima and absolute minima COULD exist even if EVT doesn't guarantee their existence.

f is not cont on $[-6, 5]$ bc

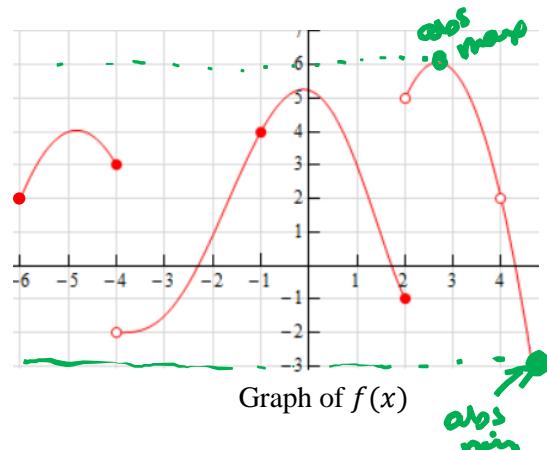
$$\lim_{x \rightarrow -4^-} f(x) \neq \lim_{x \rightarrow -4^+} f(x)$$

$$3 \neq -2$$

$$\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$$

$$-1 \neq 5$$

$f(4)$ DNE



The Extreme Value Theorem ONLY tells us that we CAN find an extreme value if a function is continuous.