

## Extreme Values of Functions

### What is an Extrema?

**Extremum** – point where the maximum (largest  $y$ -value) or minimum (smallest  $y$ -value) occurs

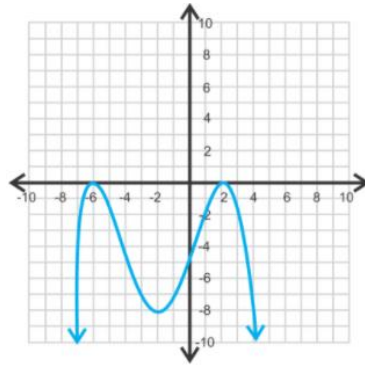
**Extrema** – points where the maxima (largest  $y$ -value) or minima (smallest  $y$ -value) occur

**Extreme Value** –  $y$ -value of the maximum or minima

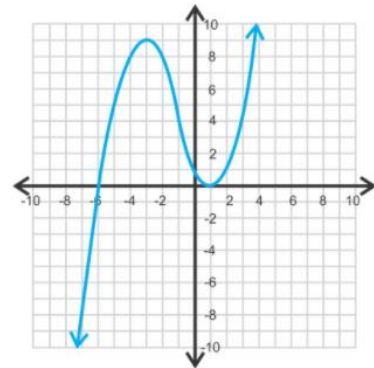
**\*Absolute Maximum Value** – THE highest  $y$ -value on entire graph or the given interval.  
(Global)

$$\text{symbolically} \rightarrow f(c) \geq f(x) \quad \forall x$$

graphically  $\rightarrow$  absolute maximum value  
on  $(-\infty, \infty)$



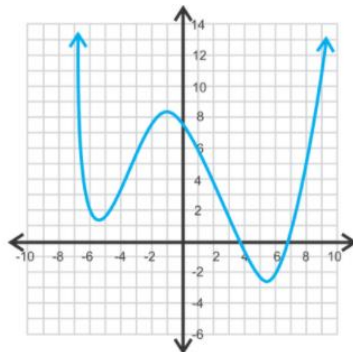
absolute maximum value  
on  $(-\infty, 3]$



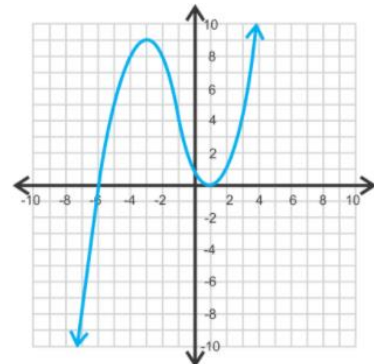
**\*Absolute Minimum Value** – THE lowest  $y$ -value on entire graph or the given interval.  
(Global)

$$\text{symbolically} \rightarrow f(c) \leq f(x) \quad \forall x$$

graphically  $\rightarrow$  absolute minimum value  
on  $(-\infty, \infty)$



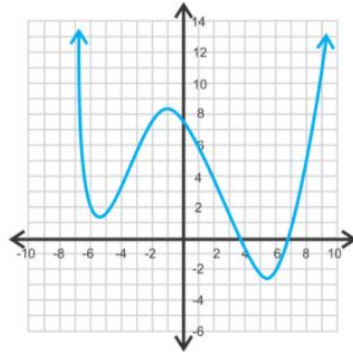
absolute minimum value  
on  $[-6, \infty)$



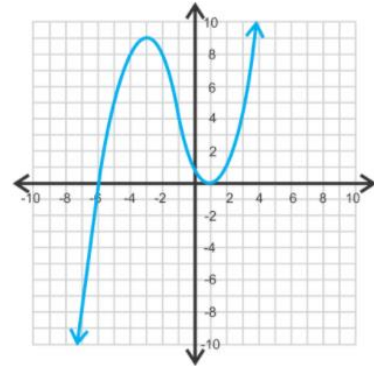
\***Relative Maximum Value** – highest  $y$ -value on an open interval. (think: where  $f(x)$  changes from increasing to decreasing)  
(Local)

symbolically  $\rightarrow f(c) \geq f(x) \quad \forall x$  on  $(a, b)$

graphically  $\rightarrow$  relative maximum value  
on  $(-\infty, \infty)$



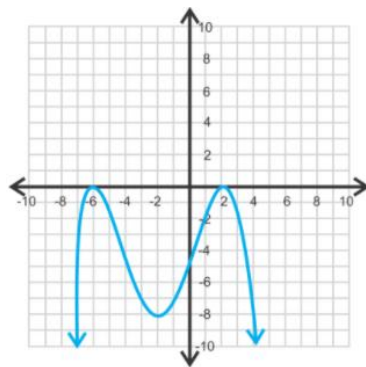
relative maximum value  
on  $(-\infty, \infty)$



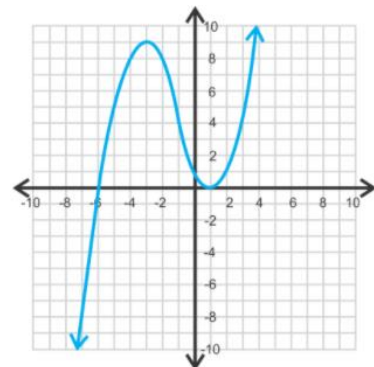
\***Relative Minimum Value** – lowest  $y$ -value on an open interval. (think: where  $f(x)$  changes from decreasing to increasing)  
(Local)

symbolically  $\rightarrow f(c) \leq f(x) \quad \forall x$  on  $(a, b)$

graphically  $\rightarrow$  relative minimum value  
on  $(-\infty, \infty)$



relative minimum value  
on  $(-\infty, \infty)$



*Example:*

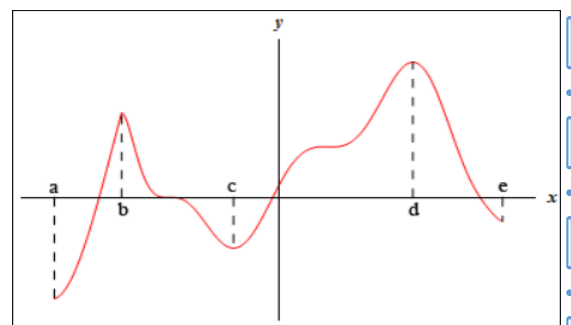
Which indicated  $x$ -value in the drawing to the right has:

a relative minimum value?

a relative maximum value?

an absolute minimum value?

an absolute maximum value?

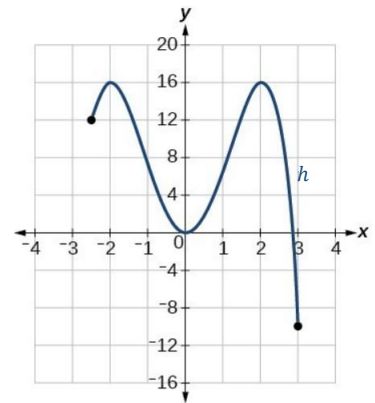
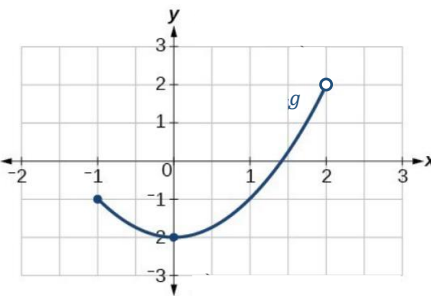
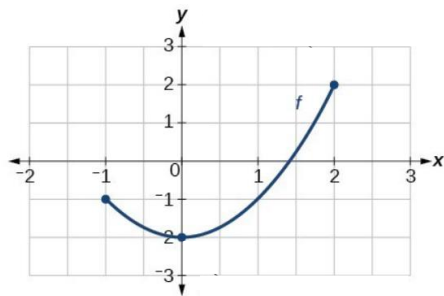


## Extreme Value Theorem (EVT)

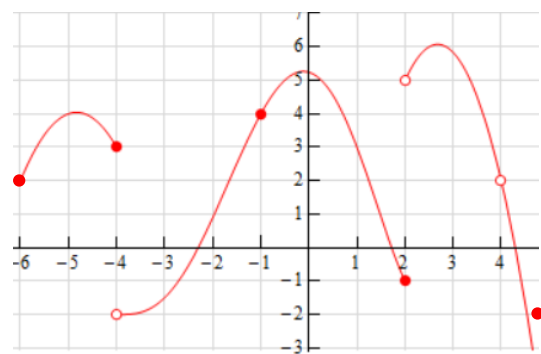
### Extreme Value Theorem

If  $f$  is continuous on  $[a, b]$ ,  
then  $f$  has both an absolute maximum and absolute minimum on that interval.  
( $f$  has extreme values)

Examples:



EVT guarantees that an absolute maximum or minimum occurs,  
but absolute maxima and absolute minima COULD exist  
even if EVT doesn't guarantee their existence



Graph of  $f(x)$

The Extreme Value Theorem ONLY tells us that we CAN find an extreme value if a function is continuous.