DATE: ______________

AP FRQs & M/C for Candidates Test (Absolute Extrema)

1. The rate at which gasoline flows out of the storage tank into trucks at time t can be modeled by the function R defined by $R(t) = \frac{100t}{t^2+4}$, where t is measured in hours, and $R(t)$ is measured in thousands of gallons. Based on the model, at what time t, for $0 \le t \le 10$, is the rate at which gasoline flows out of the storage tank an absolute maximum? Justify your answer.

$R'(t) = \dfrac{(t^2+4)(100) - 100t(2t)}{(t^2+4)^2}$

$= \dfrac{100t^2 + 400 - 200t^2}{(t^2+4)^2}$

$0 = \dfrac{400 - 100t^2}{(t^2+4)^2}$

$0 = 400 - 100t^2$

$100t^2 = 400 \longrightarrow t^2 = 4$, $t = \pm 2 \rightarrow$ crit # of only $t=2$ is in $[0,10]$

$R(2) = \dfrac{100(2)}{2^2+4} = \dfrac{200}{8} = 25$

$R(0) = 0$

$R(10) = \dfrac{100(10)}{10^2+4} = \dfrac{1000}{104} < 25$

$\boxed{\text{The rate at which gasoline flows out of the storage tank is an abs. max is @ time, } t = 2\text{hrs}}$

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2. If $f(x) = \frac{1}{3}x^3 - 4x^2 + 12x - 5$ and the domain is the set of all x such that $0 \le x \le 9$, then the absolute maximum value of the function f occurs when x is

(A) 0

(B) 2

(C) 4

(D) 6

$\boxed{\text{(E) 9}}$

$f'(x) = x^2 - 8x + 12$

$0 = x^2 - 8x + 12$

$0 = (x-6)(x-2)$

$x = 6, x = 2 \leftarrow$ crit #s

$\boxed{\text{Abs max value of } f \text{ occurs @ } x = 9}$

$f(2) = \frac{1}{3}(2)^3 - 4(2)^2 + 12(2) - 5$
$= \frac{8}{3} - 16 + 24 - 5 = \frac{8}{3} + 3 = \frac{17}{3}$

$f(6) = \frac{1}{3}(6)^3 - 4(6)^2 + 12(6) - 5$
$= 72 - 144 + 72 - 5 = -5$

$f(9) = \frac{1}{3}(9)^3 - 4(9)^2 + 12(9) - 5$
$= 9(\frac{1}{3}(9)^2 - 4(9) + 12) - 5$
$= 9(27 - 36 + 12) - 5$
$= 9(3) - 5$
$= 27 - 5 = 22$

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3. Let g be a continuous function on the closed interval [0, 1]. Let $g(0)=1$ and $g(1)=0$. Which of the following is NOT necessarily true?

(A) There exists a number h in [0, 1] such that $g(h) \ge g(x)$ for all x in [0, 1]. $\rightarrow$ abs max for g is TRUE b/c g is cont on [0, 1], so, by EVT, g would have abs max on [0, 1]

(B) For all a and b in [0, 1], if $a = b$, then $g(a) = g(b)$. $\rightarrow$ if same x-value, then same y-value

(C) There exists a number h in [0, 1] such that $g(h) = \frac{1}{2}$. $\rightarrow$ TRUE, by IVT, g is cont on [0, 1] and $g(1) < \frac{1}{2} < g(0)$, so there exists a # in [0, 1] such that $g(h) = \frac{1}{2}$

$\boxed{\text{(D)}}$ There exists a number h in [0, 1] such that $g(h) = \frac{3}{2}$.

(E) For all h in the open interval (0, 1), $\lim_{x \to h} g(x) = g(h)$.

TRUE, $\lim_{x \to h} g(x) = g(h)$ means g is cont on [0, 1]

$\rightarrow$ not necessarily TRUE b/c $g(0) \not< \frac{3}{2}$ and $g(1) < \frac{3}{2}$, so IVT does not apply.