DATE: ______________

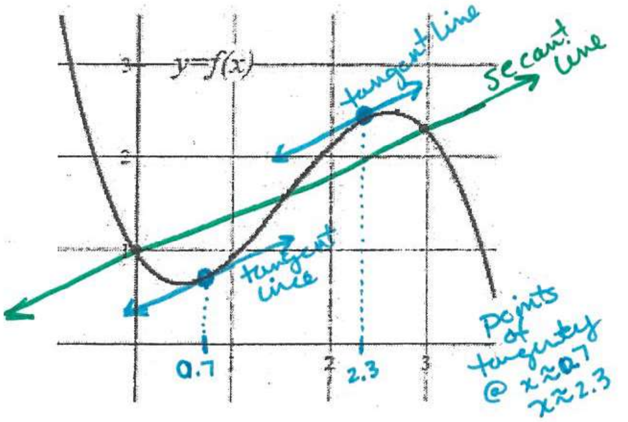
Mean Value Theorem

1.

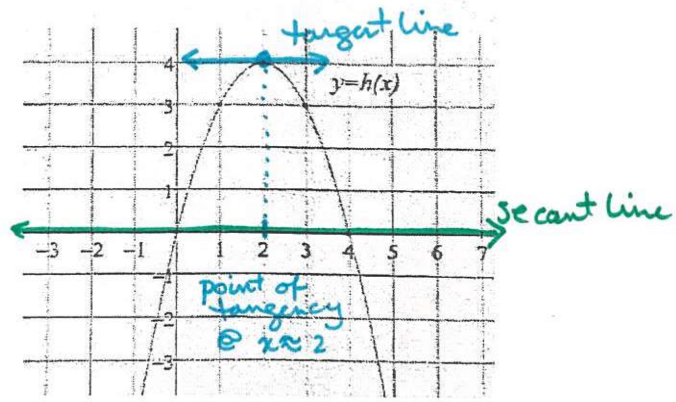
a) For each graph, draw the secant line through the two points on the graph corresponding to the endpoints on the indicated interval.

b) In the indicated interval, draw any tangent line(s) that are parallel to the secant line. Estimate the x-value of the point of tangency.

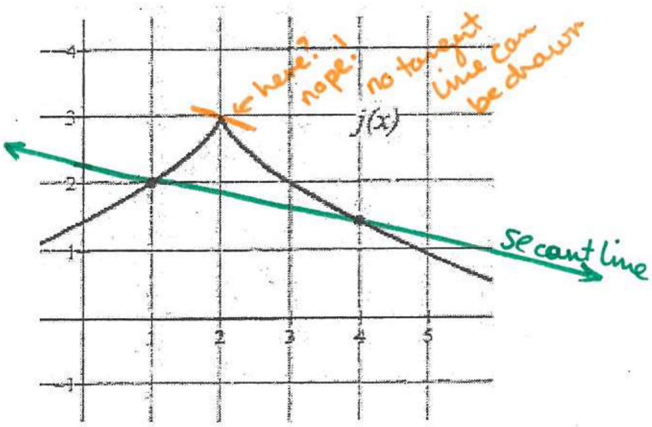
$f(x)$ on the interval $[0, 3]$



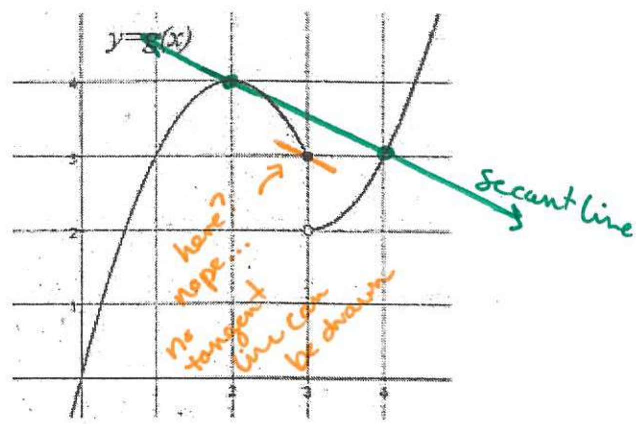
$h(x)$ on the interval $[0, 4]$



$j(x)$ on the interval $[1, 4]$



$g(x)$ on the interval $[2, 4]$



2.

a) Which graphs are continuous on $[a, b]$? f(x), h(x), j(x)

If the function is continuous on $[a, b]$, is there a tangent line parallel to the secant line? not always

b) Which graphs are differentiable on (a, b)? f(x), h(x)

If the function is differentiable on (a, b), is there a tangent line parallel to the secant line? yes!

c) What can you conclude must be true about a function in order to draw a tangent line parallel to the secant line?

A function MUST be differentiable
(and if a function is differentiable, then the function is also continuous)

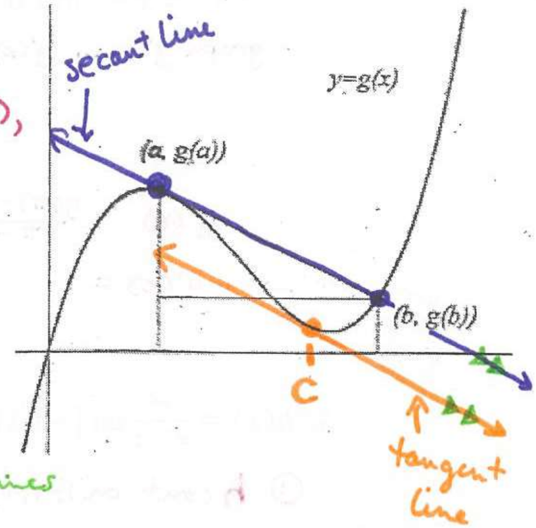
MEAN VALUE THEOREM

↳ "average rate of change" → "slope" → "derivative"

If f is cont on [a,b] and diff'able on (a,b),

then ∃ a #, c, in (a,b) s.t.

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$



This means:

① f cont on [a,b] → no jumps, holes, or V.A.

② f diff'able on (a,b) → no corner, no discont, no vertical tangent lines

③ f'(c) = slope of line for (a, f(a)) + (b, f(b))

(slope of tangent line @ x = c = slope of secant line through (a, f(a)) & (b, f(b)))

(tangent line is parallel to secant line)

Examples:

Determine if the Mean Value Theorem applies. If it does apply, explain what conclusions you can draw from it; if it does not apply, state why not.

1. $f(x) = x^3 - x^2 - 2x$ on $[-1,1]$

① f cont on [-1,1]? yes b/c f(x) is polynomial

② f diff'able on (-1,1)? yes. b/c f(x) is polynomial

} ∴, MVT applies

③ $f'(x) = 3x^2 - 2x - 2$

$$\frac{f(b) - f(a)}{b - a} = \frac{f(1) - f(-1)}{1 - -1} = \frac{-2 - 0}{2} = -1$$

$$3c^2 - 2c - 2 = -1$$

$$3c^2 - 2c - 1 = 0$$

$$(3c+1)(c-1) = 0$$

$c = -\frac{1}{3}$, $\cancel{c = 1}$ ... so $f'(-\frac{1}{3}) = -1$

not in open interval (-1, 1)

(slope of tangent line is −1 @ $x = -\frac{1}{3}$) ... ☺

use calculator

2. $g(x) = x - \sin x$ on $[-\pi, \pi]$

graph $g'(x)$ ... $g'(x)$ exists on $[-\pi, \pi]$, $\therefore$ $g(x)$ diff'able on $[-\pi, \pi]$

① g cont on $[-\pi, \pi]$? yes b/c g diff'able on $[-\pi, \pi]$

② g diff'able on $(-\pi, \pi)$? yes b/c graph of $g'(x)$ exists on $(-\pi, \pi)$

$\therefore$, MVT applies

③ $g'(x) = \frac{g(\pi) - g(-\pi)}{\pi - -\pi}$

use calculator... find intersection

$g'(c) = 1$

$c = -1.571$, $c = 1.571$

$\therefore$ $g'(1.571) = 1$

$g'(-1.571) = 1$

3. $h(x) = \frac{x^2}{x^2-1}$ on $[-1,1]$.

① h cont on $[-1,1]$? h discontinuous when $x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

so, h is not cont on $[-1,1]$

$\therefore$, MVT does not apply.