4.2 Trigonometric Functions of Acute Angles

Target 5B: Generate the unit circle from special right triangles

Review of Prior Concepts

1. Convert each radian measure to degrees:

a) $\frac{\pi}{6}$

$\left(\frac{\cancel{\pi}}{6}\ \cancel{\text{radian}}\right)\left(\frac{180^\circ}{\cancel{\pi}\ \cancel{\text{radian}}}\right)$

$\boxed{30^\circ}$

b) $\frac{\pi}{4}$

$\left(\frac{\cancel{\pi}}{4}\ \cancel{\text{radian}}\right)\left(\frac{180^\circ}{\cancel{\pi}\ \cancel{\text{radian}}}\right)$

$\boxed{45^\circ}$

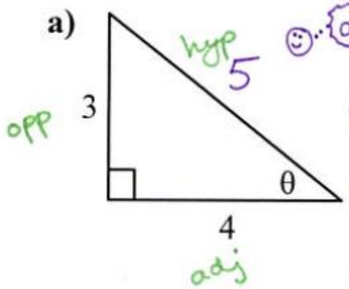
c) $\frac{\pi}{3}$

$\left(\frac{\cancel{\pi}}{3}\ \cancel{\text{radian}}\right)\left(\frac{180^\circ}{\cancel{\pi}\ \cancel{\text{radian}}}\right)$

$\boxed{60^\circ}$

2. Find the values of $\sin\theta$, $\cos\theta$, $\tan\theta$.

a)



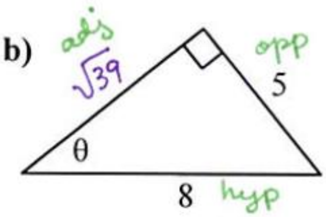
☺: {$a^2+b^2=c^2$}

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$\tan\theta = \frac{3}{4}$

b)



$a^2+5^2=8^2$

$a^2+25=64$

$\sqrt{a^2}=\sqrt{39}$

$\sin\theta = \frac{5}{8}$

$\cos\theta = \frac{\sqrt{39}}{8}$

$\tan\theta = \frac{8}{\sqrt{39}}$

☺: {rationalize optional}

More Practice

Trigonometry

https://www.khanacademy.org/math/trigonometry/trigonometry-right-triangles
http://www.mathsisfun.com/algebra/trigonometry.html
http://www.regentsprep.org/regents/math/algebra/at2/ltrig.htm
http://www.mathgoodies.com/lessons/vol2/circumference.html
https://www.youtube.com/watch?v=SqFQZWRALGc
https://www.youtube.com/watch?v=Jsiy4TxgIME

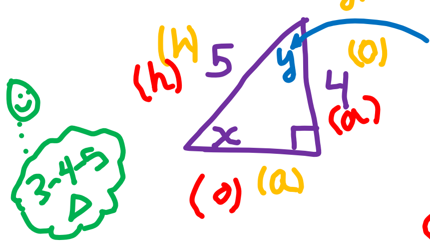
SAT Connection

Passport to Advanced Math

14. Use structure to isolate or identify a quantity of interest in an expression

Example: In a right triangle, one angle measures x°, where $\sin x^\circ = \frac{4}{5}$. What is $\cos(90^\circ - x^\circ)$?

opposite / hypotenuse



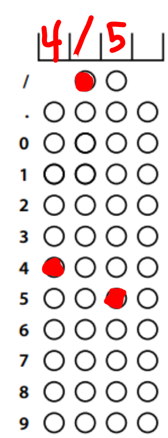
$\angle y + \angle x + 90 = 180$

$\angle y = 180 - 90 - x$

$\angle y = 90 - x$

$\cos(90 - x) = \cos y$

$= \frac{\text{adj}}{\text{hyp}} = \frac{4}{5}$



NOTE: You may start your answers in any column, space permitting. Columns you don't need to use should be left blank.

Solution

Six Trigonometric Ratios

Six Trigonometric Ratios

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$

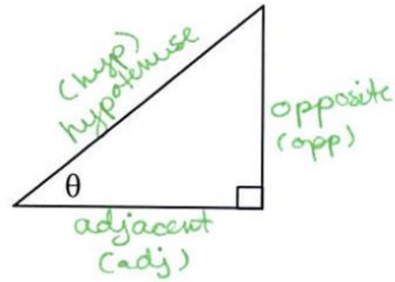
$\cos\theta = \frac{\text{adj}}{\text{hyp}}$

$\tan\theta = \frac{\text{opp}}{\text{adj}}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{1}{\sin\theta}$

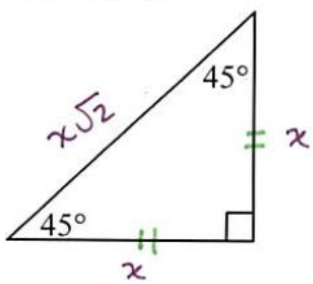
$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{1}{\cos\theta}$

$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{1}{\tan\theta}$



Special Right Triangles

45°–45°–90° Δ



$x^2 + x^2 = c^2$

$2x^2 = c^2$

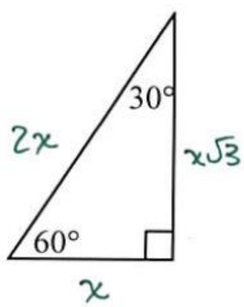
$\sqrt{2x^2} = c$

$x\sqrt{2} = c$

What do you know about a 45°–45°–90° Δ?

Sides are: $x, x, x\sqrt{2}$

30°–60°–90° Δ



$a^2 + x^2 = (2x)^2$

$a^2 + x^2 = 4x^2$

$a^2 = 3x^2$

$a = \sqrt{3x^2}$

$a = x\sqrt{3}$

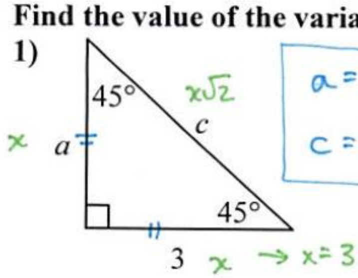
What do you know about a 30°–60°–90° Δ?

Sides are: $x, x\sqrt{3}, 2x$

Examples

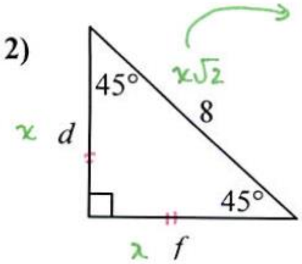
Find the value of the variables.

1)



$\boxed{a = 3 \quad c = 3\sqrt{2}}$

2)



$8 = x\sqrt{2}$

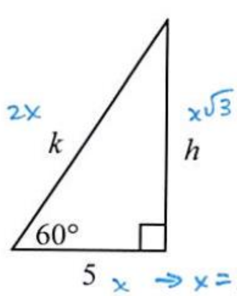
$\frac{8}{\sqrt{2}} = x$

$\frac{8\sqrt{2}}{2} = x$

$4\sqrt{2} = x$

$\boxed{d = 4\sqrt{2} \quad f = 4\sqrt{2}}$

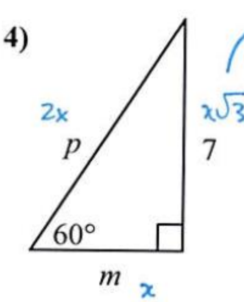
3)



$k = 2(5)$

$\boxed{k = 10 \quad h = 5\sqrt{3}}$

4)



$7 = x\sqrt{3}$

$\frac{7}{\sqrt{3}} = x$

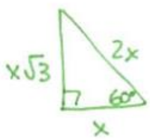
$\frac{7\sqrt{3}}{3} = x$

$\boxed{m = \frac{7\sqrt{3}}{3} \quad p = \frac{14\sqrt{3}}{3}}$

Evaluate without using a calculator:

5) $\tan\left(\frac{\pi}{3}\right)$ $\frac{\pi}{3}\cdot\frac{180^\circ}{\pi}=60^\circ$

$\tan(60^\circ)$



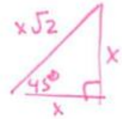
$\tan 60^\circ = \frac{opp}{adj}$

$= \frac{x\sqrt{3}}{x}$

$\boxed{\tan\left(\frac{\pi}{3}\right)=\sqrt{3}}$

6) $\csc\left(\frac{\pi}{4}\right)$ $\frac{\pi}{4}\cdot\frac{180^\circ}{\pi}=45^\circ$

$\csc(45^\circ)$



$\csc(45^\circ)=\frac{hyp}{opp}$

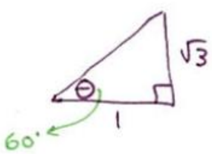
$=\frac{x\sqrt{2}}{x}$

$\boxed{\csc\left(\frac{\pi}{4}\right)=\sqrt{2}}$

Find the acute angle θ, in both degrees and radians, without using a calculator.

7) $\tan\theta=\sqrt{3}$ which △ uses $\sqrt{3}$? 30°-60°-90° △

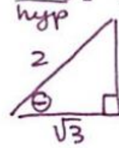
$\frac{opp}{adj}=\frac{\sqrt{3}}{1}$



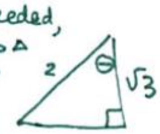
$\boxed{\theta=60^\circ \text{ or } \frac{\pi}{3}}$

8) $\cos\theta=\frac{\sqrt{3}}{2}$ which △ uses $\sqrt{3}$? 30°-60°-90°

$\frac{adj}{hyp}=\frac{\sqrt{3}}{2}$



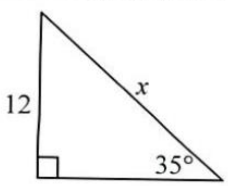
if needed, flip △



$\boxed{\theta=30^\circ \text{ or } \frac{\pi}{6}}$

Find the value of x in the triangle.

9)



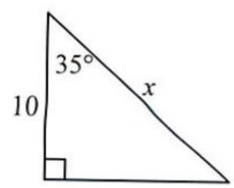
$\sin 35^\circ=\frac{12}{x}$

$x\sin 35^\circ=12$

$x=\frac{12}{\sin 35^\circ}$

$\boxed{x=20.921}$

10)



$\cos 35^\circ=\frac{10}{x}$

$x\cos 35^\circ=10$

$x=\frac{10}{\cos 35^\circ}$

$\boxed{x=12.208}$

More Practice

Special Right Triangles

http://www.regentsprep.org/regents/math/algtrig/att2/ltri45.htm
http://www.regentsprep.org/regents/math/algtrig/att2/ltri30.htm
https://www.khanacademy.org/math/trigonometry/trigonometry-right-triangles/trig-ratios-special-triangles/a/trig-ratios-of-special-triangles
https://www.youtube.com/watch?v=Wy_e8QANH_g
https://www.youtube.com/watch?v=2mlsvpox9sI

Trigonometric Ratios

http://www.regentsprep.org/regents/math/algtrig/att1/trigsix.htm
http://www.themathpage.com/atrig/solve-right-triangles.htm
http://www.mathguide.com/lessons/RightTriTrig.html
https://www.youtube.com/watch?v=l5VbdqRjTXc

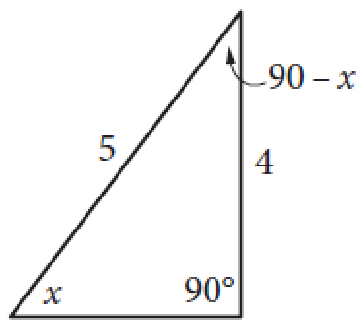
Homework Assignment

p.368 #7,19,22,41,44,47,49,51, 53,61,65

SAT Connection
Solution

The correct answer is $\frac{4}{5}$ or 0.8. By the complementary angle relationship for sine and cosine, $\sin(x°) = \cos(90° - x°)$. Therefore, $\cos(90° - x°) = \frac{4}{5}$. Either the fraction $\frac{4}{5}$ or its decimal equivalent, 0.8, may be gridded as the correct answer.

Alternatively, one can construct a right triangle that has an angle of measure $x°$ such that $\sin(x°) = \frac{4}{5}$, as shown in the figure below, where $\sin(x°)$ is equal to the ratio of the opposite side to the hypotenuse, or $\frac{4}{5}$.



Since two of the angles of the triangle are of measure $x°$ and $90°$, the third angle must have the measure $180° - 90° - x° = 90° - x°$. From the figure, $\cos(90° - x°)$, which is equal to the ratio of the adjacent side to the hypotenuse, is also $\frac{4}{5}$.