DATE: ____________

MVT AP Practice Problems

AP F/R Calculator Problem

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

1. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24-hour period. Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

R(t) diff'able so R(t) is also cont.

$$\left.\begin{matrix} R(0) = 9.6 \\ R(24) = 9.6 \end{matrix}\right\} \quad R'(c) = \frac{R(24) - R(0)}{24 - 0} = \frac{9.6 - 9.6}{24} = 0$$

MVT Justification

By MVT, there is some time t, on (0, 24) s.t. $R'(t) = 0$. answer

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AP M/C Non-Calculator Problems

2. Let f be a polynomial function with degree greater than 2. If $a \neq b$ and $f(a) = f(b) = 1$, which of the following must be true for at least one value of x between a and b?

I. $f(x) = 0$
II. $f'(x) = 0$
III. $f''(x) = 0$

(A) None **(B)** I only $\boxed{\textbf{(C)}}$ II only **(D)** I and II only **(E)** I, II, and III

II. $f'(x) = 0$ TRUE b/c of MVT

f cont & diff'able b/c polynomial

$$f'(x) = \frac{f(b) - f(a)}{b - a} = \frac{1 - 1}{b - a}$$

$f'(x) = 0$

I. False by IVT only guarantees $f(x) = 0$ if $f(a)$ & $f(b)$ have values where one is greater than zero & one is less than zero. But both $f(a)$ & $f(b)$ are greater than zero, so $f(x)$ doesn't have to = zero.

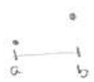
3. Let f be a polynomial function where $f(b) > f(a)$. Which of the following is true for at least one value of x on the interval (a, b)?

I. The function f is differentiable on (a, b)
II. There exists a number k on (a,b) such that $f'(k) < 0$
III. There exists a number k on (a,b) such that $f'(k) > 0$

(A) I only **(B)** II only **(C)** I and II $\boxed{\textbf{(D)}}$ I and III **(E)** I, II, and III

✓ I. All poly's are diff'able

II or III



$f'(k) = \frac{f(b) - f(a)}{b - a} = \frac{+\ \#}{+\ \#} = +$

$f'(k) > 0$

↳ so III true.

$f(x) = 3\sqrt{x} + 1$

4. Which of the following statements is true for $f(x) = \sqrt[3]{x} + 1$?

I. $f(x)$ is always increasing, $x \neq 0$.
II. The tangent to the curve at $x = 0$ is horizontal.
III. The Mean Value Theorem can be applied to $f(x)$ in the closed interval $-1 \leq x \leq 1$.

$\boxed{\textbf{(A)}}$ I only **(B)** II only **(C)** III only **(D)** II and III **(E)** I, II, and III

I. $f'(x) = \frac{1}{3}x^{-2/3}$

$f'(x)$ DNE @ $x = 0$ $f'(x)$ + | + (−1) 0 (1)

$f(x)$ always increasing

II. tangent line @ $x = 0$ ↳ VERTICAL

III. MVT cannot be applied b/c $f(x)$ not diff'able @ $x = 0$

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5. Find a positive value c, for x, that satisfies the conclusion of the Mean Value Theorem for Derivatives of $f(x) = 3x^2 - 5x + 1$ on the interval [2,5].

$f'(x) = 6x - 5$

(A) 1 **(B)** 13/6 **(C)** 11/6 **(D)** 23/6 $\boxed{\textbf{(E)}}$ 7/2

$f(5) = 3(5)^2 - 5(5) + 1 = 51$

$f(2) = 3(2)^2 - 5(2) + 1 = 3$

$f'(c) = \frac{51 - 3}{5 - 2}$

$6c - 5 = \frac{48}{3}$

$6c - 5 = 16$

$6c = 21$

$c = \frac{21}{6}$

$c = \frac{7}{2}$