DATE: ____________

Test for Concavity M/C Practice

1. If $f''(x) = (x-1)(x+2)^3(x-4)^2$, then the graph of f has inflection points when $x =$

(A) -2 only

(B) 1 only

(C) 1 and 4 only

(D) -2 and 1 only

(E) -2, 1, and 4 only

$f''=0$ @ $x=1$, $x=-2$, $x=4$

f'': $+$ | $-$ | $+$ | $+$ (test points -3, -2, 0, 1, 2, 4, 5)

f has inf. pts @ $x=-2$ and $x=1$ b/c f'' changes signs @ $x=-2$ and $x=1$

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2. The function $f(x) = xe^x$ has inflection points at:

(A) -2

(B) -1

(C) 0

(D) 1

(E) There are no inflection point of f.

$f'(x) = e^x(1) + xe^x$
$= e^x + xe^x$
$f''(x) = e^x + e^x + xe^x$
$0 = 2e^x + xe^x$
$0 = e^x(2+x)$
$\cancel{e^x = 0}$ $x=-2$

f'': $-$ | $+$ (test points -4, -2, 0)

f has inf pt @ $x=-2$ b/c f'' changes signs @ $x=-2$.

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3. The number of inflection points of $f(x) = 3x^7 - 10x^5$ is:

(A) 0

(B) 1

(C) 2

(D) 3

(E) 5

$f'(x) = 21x^6 - 50x^4$
$f''(x) = 126x^5 - 200x^3$
$0 = 126x^5 - 200x^3$
$0 = 2x^3(63x^2 - 100)$
$0 = 2x^3(\sqrt{63}x - 10)(\sqrt{63}x + 10)$
$x = 0$, $x = \frac{10}{\sqrt{63}}$, $x = \frac{-10}{\sqrt{63}}$

f'': $-$ | $+$ | $-$ | $+$ (test points $-\sqrt{63}$, $\frac{-10}{\sqrt{63}}$, -1, 0, 1, $\frac{10}{\sqrt{63}}$, $\sqrt{63}$)

f has 3 inf. pts b/c f'' changes signs three times.

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4. For which of the following intervals is the graph of $y = x^4 - 2x^3 - 12x^2$ concave down?

(A) $(-2, 1)$

(B) $(-1, 2)$

(C) $(-2, -1)$

(D) $(-\infty, -1)$

(E) $(-1, \infty)$

$y'' < 0$

$y' = 4x^3 - 6x^2 - 24x$
$y'' = 12x^2 - 12x - 24$
$0 = 12(x^2 - x - 2)$
$0 = 12(x-2)(x+1)$
$x = 2$, $x = -1$

y'': $+$ | $-$ | $+$ (test points -4, -1, 0, 2, 5)

y is concave down on $(-1, 2)$ b/c $y'' < 0$ on $(-1, 2)$