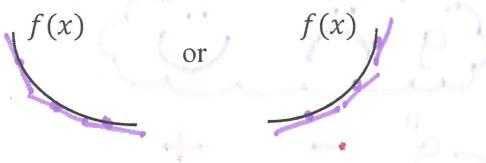
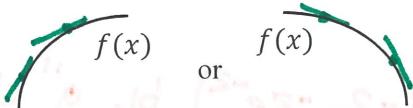


4.3 Connecting f' and f'' with the Graph of f **CONCAVITY - curve of graph*** f is concave up f is concave up $\rightarrow f'$ increasing $\rightarrow f'' > 0$ * f is concave down f is concave down $\rightarrow f'$ decreasing $\rightarrow f'' < 0$ Test for Concavity f'' must exist!

- ① Find where $f'' = 0$ or f'' DNE (**possible inflection pts**
p.i.p's)
- ② If $f'' > 0$, then f is **concave up**

If $f'' < 0$, then f is **concave down**

- ③ Inflection points occur where concavity changes (**f'' change signs**)

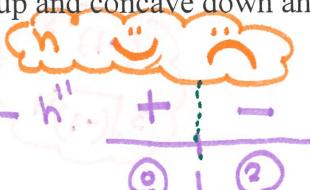
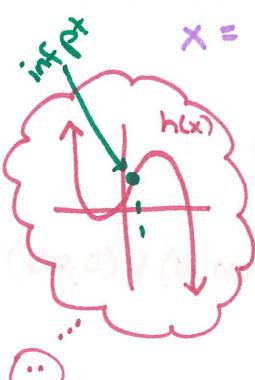
*Example 1*Find where $h(x) = 3x^2 - x^3$ is concave up and concave down and the inflection point(s) of $h(x)$ (if any).

$$h'(x) = 6x - 3x^2$$

$$h''(x) = 6 - 6x$$

$$6 - 6x = 0$$

$$x = 1 \leftarrow \text{poss.inf.pt} \quad (\text{p.i.p})$$



- make h'' # line
 - put p.i.p's on # line
 - test # s
- $\hookrightarrow h'' > 0 \rightarrow h$ conc up
 $h'' < 0 \rightarrow h$ conc down

h has inf. pts @ $(1, 2)$ b/c h'' # changes signs
 $\quad \quad \quad @ x = 1$

h is concave down on $(1, \infty)$ b/c $h'' < 0$ on $(1, \infty)$

h is concave up on $(-\infty, 1)$ b/c $h'' > 0$ on $(-\infty, 1)$

Example 2

Find where $g(x) = xe^x$ is concave up and concave down and the inflection point(s) of $g(x)$ (if any).

$$g'(x) = e^x(1) + xe^x$$

$$\begin{aligned} g''(x) &= e^x + e^x(1) + xe^x \\ &= 2e^x + xe^x \end{aligned}$$

$$\begin{aligned} 0 &= 2e^x + xe^x \\ 0 &= e^x(2+x) \end{aligned}$$

$$e^x = 0 \quad 2+x = 0$$

$$x = -2 \leftarrow \text{p.p.}$$



$$g'' \begin{array}{c} - \\ \hline (-3) \end{array} \begin{array}{c} + \\ \hline -2 \end{array} \begin{array}{c} \circ \\ \hline 0 \end{array}$$

g has inf pt @ $(-2, -2e^{-2})$ b/c g'' changes signs @ $x = -2$.

g is concave up on $(-2, \infty)$ b/c $g'' > 0$ on $(-2, \infty)$

g is concave down on $(-\infty, -2)$ b/c $g'' < 0$ on $(-\infty, -2)$

Example 3

Find where $f(x) = x^{2/3}$ is concave up and concave down and the inflection point(s) of $f(x)$ (if any).

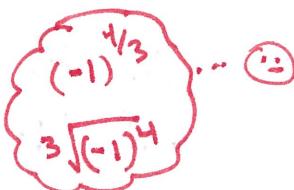
$$f'(x) = \frac{2}{3}x^{-1/3}$$

$$f''(x) = -\frac{2}{9}x^{-4/3}$$

$$-\frac{2}{9}x^{-4/3} \Rightarrow \text{DNE}$$

$$-\frac{2}{9}x^{-4/3} \text{ DNE } @ x = 0$$

$$f'' \begin{array}{c} - \\ \hline (-1) \end{array} \begin{array}{c} + \\ \hline 0 \end{array} \begin{array}{c} - \\ \hline 1 \end{array}$$



f does not have any inf. pts b/c f'' never changes signs.

f is concave down on $(-\infty, 0) \cup (0, \infty)$ b/c $f'' < 0$ on those intervals.

f is never concave up b/c $f'' < 0$ on $(-\infty, 0) \cup (0, \infty)$