2^{nd} Derivative Test (for finding max/min)

Recall:
1ST Derivative Test
finds max/min,
so does
2nd derivative Test
... ☺

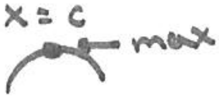
If $f'(c)=0$ and $f''(x)$ exists,
(crit # @ $x=c$)

① If $f''(c)>0$, then $f(c)$ is a rel. min

f concave up
@ crit #,
$x=c$

② If $f''(c)<0$, then $f(c)$ is a rel. max

f concave down
@ crit #,
$x=c$



③ If $f''(c)=0$, then test fails. Try 1^{st} derivative test instead.

Find rel max/min using 2nd derivative Test

ex: $f(x) = 2x^3 + 3x^2 - 12x$

$f'(x) = 6x^2 + 6x - 12$

$0 = 6(x^2 + x - 2)$

$0 = 6(x+2)(x-1)$

$\underbrace{x = -2,\ x = 1}_{\text{crit \#s}}$

2nd derivative Test

$f''(x) = 12x + 6$

$f''(1) = 18 > 0$, so rel. min @ $(1, -7)$ b/c $f''(1) > 0$

$f''(-2) = -18 < 0$, so rel. max @ $(-2, 20)$ b/c $f''(-2) < 0$

ex: $f(x) = \frac{x^2+1}{x}$

$= x + \frac{1}{x}$

$f'(x) = 1 - x^{-2}$

$0 = 1 - x^{-2}$

$\frac{1}{x^2} = 1$

$x^2 = 1$

$x = \pm 1$ crit #s

$f''(x) = 2x^{-3}$

$f''(1) = 2 > 0$, so rel. min @ $(1, 2)$ b/c $f''(1) > 0$

$f''(-1) = 2(-1)^{-3} < 0$, so rel. max @ $(-1, -2)$ b/c $f''(-1) < 0$

ex: $f(x) = x^2(6-x)^3$

$$f'(x) = (6-x)^3(2x) + x^2\left[3(6-x)^2(-1)\right]$$

$$= 2x(6-x)^3 - 3x^2(6-x)^2$$

$$= x(6-x)^2\left[2(6-x) - 3x\right]$$

$$= x(6-x)^2(12-2x-3x)$$

$$0 = x(6-x)^2(12-5x) \rightarrow (6-x)^2(12x-5x^2)$$

$$\underbrace{x=0,\ x=6,\ x=12/5}_{\text{crit \#s}}$$

2nd derivative Test

$$f''(x) = (12x-5x^2)\left[2(6-x)(-1)\right] + (6-x)^2(12-10x)$$

$$= -2(6-x)(12x-5x^2) + (6-x)^2(12-10x)$$

$$= 2(6-x)\left[-(12x-5x^2) + (6-x)(6-5x)\right]$$

$$= 2(6-x)\left[-12x+5x^2+36-6x-30x+5x^2\right]$$

$$= 2(6-x)(10x^2-48x+36)$$

$f''(0) = 2(6)(36) > 0$, so rel. min @ $(0,0)$ b/c $f''(0) > 0$

$f''(6) = 2(0)(\#) = 0 \rightarrow$ 2nd derivative test FAILS (need to do 1st deriv. test)

$f''(12/5) = 2(+)(-) < 0$, so rel. max @ $\left(\frac{12}{5}, 268.739\right)$ b/c $f''(12/5) < 0$

1st deriv Test

$$f' \quad \begin{array}{ccc} - & & - \\ \hline ⑤ & 6 & ⑦ \end{array}$$

$\rightarrow$ no rel. max/min @ $x=6$ b/c f' doesn't change signs