

DATE: _____

Curve Sketching

Suppose that you have the following information about a point $(a, f(a))$ on the graph of a continuous function $f(x)$. What do you know about the point $(a, f(a))$?

a) $f'(a) = 0$

$(a, f(a))$ is a critical point

b) $f'(a) = 0$, and $f'(x) < 0$ on (b, a) , and $f'(x) > 0$ on (a, c) , where $b < a < c$.

$$\begin{array}{c} f' \\ \leftarrow - + \rightarrow \\ b \quad a \quad c \end{array}$$

$(a, f(a))$ is a rel. min b/c $f'(x)$ changes from neg to pos @ $x=a$

c) $f'(a) = 0$ and $f''(a) > 0$

$(a, f(a))$ is a critical pt + $\underbrace{f''(a) > 0}_{f \text{ concave up @ } x=a}$

$\therefore, (a, f(a))$ is a rel. min by 2nd derivative test ☺

d) $f''(a) = 0$

$(a, f(a))$ is a possible inflection point.

e) $f''(a) = 0$, and $f''(x) < 0$ on (b, a) , and $f''(x) < 0$ on (a, c) , where $b < a < c$.

$$\begin{array}{c} f'' \\ \leftarrow - + - \rightarrow \\ b \quad a \quad c \end{array}$$

$(a, f(a))$ is a possible inf pt. but $(a, f(a))$ is not actually an inflection pt b/c f'' does not change signs @ $x=a$.