



Let  $f$  be a twice-differentiable function defined on the interval  $-1.2 < x < 3.2$  with  $f(1) = 2$ . The graph of  $f'$ , the derivative of  $f$ , is shown above. The graph of  $f'$  crosses the  $x$ -axis at  $x = -1$  and  $x = 3$  and has a horizontal tangent at  $x = 2$ . Let  $g$  be the function given by  $g(x) = e^{f(x)}$ .

- (a) Write an equation for the line tangent to the graph of  $g$  at  $x = 1$ .
- (b) For  $-1.2 < x < 3.2$ , find all values of  $x$  at which  $g$  has a local maximum. Justify your answer.
- (c) The second derivative of  $g$  is  $g''(x) = e^{f(x)} [(f'(x))^2 + f''(x)]$ . Is  $g''(-1)$  positive, negative or zero? Justify your answer.
- (d) Find the average rate of change of  $g'$ , the derivative of  $g$ , over the interval  $[1, 3]$ .

a)  $y - y_1 = m(x - x_1)$        $g(1) = e^{f(1)} = e^2$        $g'(x) = f'(x) \cdot e^{f(x)}$   
 $y - g(1) = g'(1)(x - 1)$        $g'(1) = f'(1) \cdot e^{f(1)} = -4e^2$   
 $y - e^2 = -4e^2(x - 1)$

b)  $g$  has a local max @  $x = -1$  b/c  $f'$  changes from pos to neg @  $x = -1$

c)  $g''(-1) = e^{f(-1)} [(f'(-1))^2 + f''(-1)]$   
 $= e^{f(-1)} (0^2 + f''(-1))$   
 $= (+)(-)$  ... ☹️

$e^{f(-1)} > 0$  b/c  $e^x > 0 \forall x$ .  
 $f''(-1) < 0$  b/c  $f'$  dec @  $x = -1$ .

$\therefore g''(-1) < 0$

d) avg rate of change of  $g'$  =  $\frac{g'(3) - g'(1)}{3 - 1} = \frac{0 - (-4e^2)}{2} = \frac{4e^2}{2} = \frac{2e^2}{1}$

$\frac{2e^2}{2}$

$g'(3) = f'(3) \cdot e^{f(3)}$   
 $= 0 \cdot e^{f(3)}$   
 $= 0$