

4.4 Graphs of Sinusoidal Functions

Target 5E: Rigid and non-rigid transformations of sinusoids

Review of Prior Concepts

From the parent function $f(x) = x^2$, describe the transformation of $g(x) = (x - 1)^2 + 3$ and give the domain and range of $g(x)$.

vertical shift up 3 units,
horizontal shift right 1 unit

Domain: $(-\infty, \infty)$

Range: $[3, \infty)$

More Practice

Transformations

<http://www.regentsprep.org/regents/math/algtrig/atp9/funclesson1.htm>

<https://www.mathsisfun.com/sets/function-transformations.html>

<https://www.khanacademy.org/math/algebra2/manipulating-functions/stretching-functions/v/shifting-and-reflecting-functions>

<https://academics.utep.edu/Portals/1788/CALCULUS%20MATERIAL/1%20TRANSFORMATION%20OF%20FNS.pdf>

<https://www.youtube.com/watch?v=0a-AjP4UdnY>

<https://www.youtube.com/watch?v=3Q5Sy034fok>

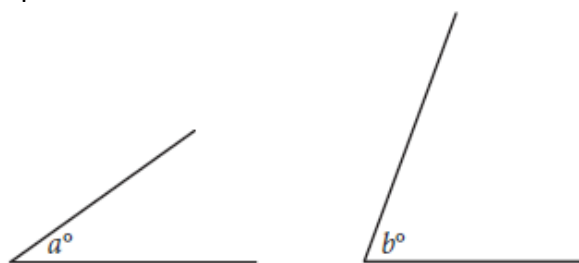


SAT Connection

Passport to Advanced Math

14. Use structure to isolate or identify a quantity of interest in an expression

Example:



Note: Figures not drawn to scale.

Since $\sin(a) = \cos(b)$

$$a = 90 - b$$

$$4k - 22 = 90 - (6k - 13)$$

$$4k - 22 = 90 - 6k + 13$$

$$4k - 22 = -6k + 103$$

$$10k = 125$$

$$k = 12.5$$



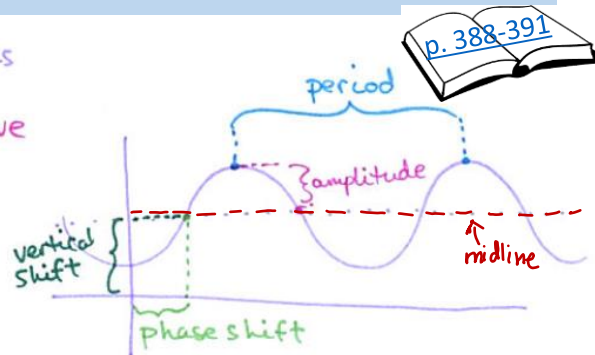
The angles shown above are acute and $\sin(a^\circ) = \cos(b^\circ)$. If $a = 4k - 22$ and $b = 6k - 13$, what is the value of k ?

- A) 4.5
- B) 5.5
- C) 12.5
- D) 21.5

[Solution](#)

Vocabulary

- Sinusoidal Functions – sine and cosine functions
- Amplitude – $\frac{1}{2}$ height of the sine/cosine wave
- Period – one cycle of the sine wave
- Phase Shift – horizontal (left/right) shift
- Vertical Shift – up/down shift
- Midline – horizontal axis through the middle y-value



TI-Nspire Activity

Open the TI-Nspire document: *Basic_Trigonometric_Transformations.tns*

Move to page 1.2

1. Drag the sliders to change the values of a in the function $f(x) = a \sin(bx)$.

a) How does the value of a affect the shape of the graph?

vertical shrink or stretch

b) What happens to the graph if a is negative?

reflects over the x-axis

c) How does the value of b affect the shape of the graph?

horizontal shrink or stretch

Conclusion:

For $a \neq 0$ and $b > 0$, the graph of $f(x) = a \sin(bx)$ has an amplitude of $|a|$ and a period of $\frac{2\pi}{|b|}$.

Move to page 2.2

2. Drag the sliders to change the value of d in the function of $f(x) = \sin(x) + d$.

How does the value of d affect the shape of the graph?

vertical shift (up/down)

Conclusion:

The graph of $f(x) = \sin(x) + d$ has a vertical shift of \underline{d} .

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3. Drag the sliders to change the value of c in the function of $f(x) = \sin(x + c)$.

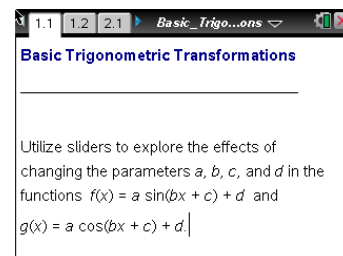
How does the value of c affect the shape of the graph?

horizontal (left/right) shift

Conclusion:

The graph of $f(x) = \sin(x + c)$ has a phase shift of \underline{c} .

$c > 0$, shift left
 $c < 0$, shift right



Move to page 4.2

4. Drag the sliders to change the value of a , b , c and d in the function $f(x) = a \sin(bx + c) + d$.
Which of the four parameters have an impact on the phase shift of the graph?

c and b

$$a \sin\left(b\left(x + \frac{c}{b}\right)\right) + d$$

↓
Phase Shift

Conclusion:

The graph of $f(x) = a \sin(bx + c) + d$ has a phase shift of $\frac{c}{b}$.

Apply Knowledge from Activity

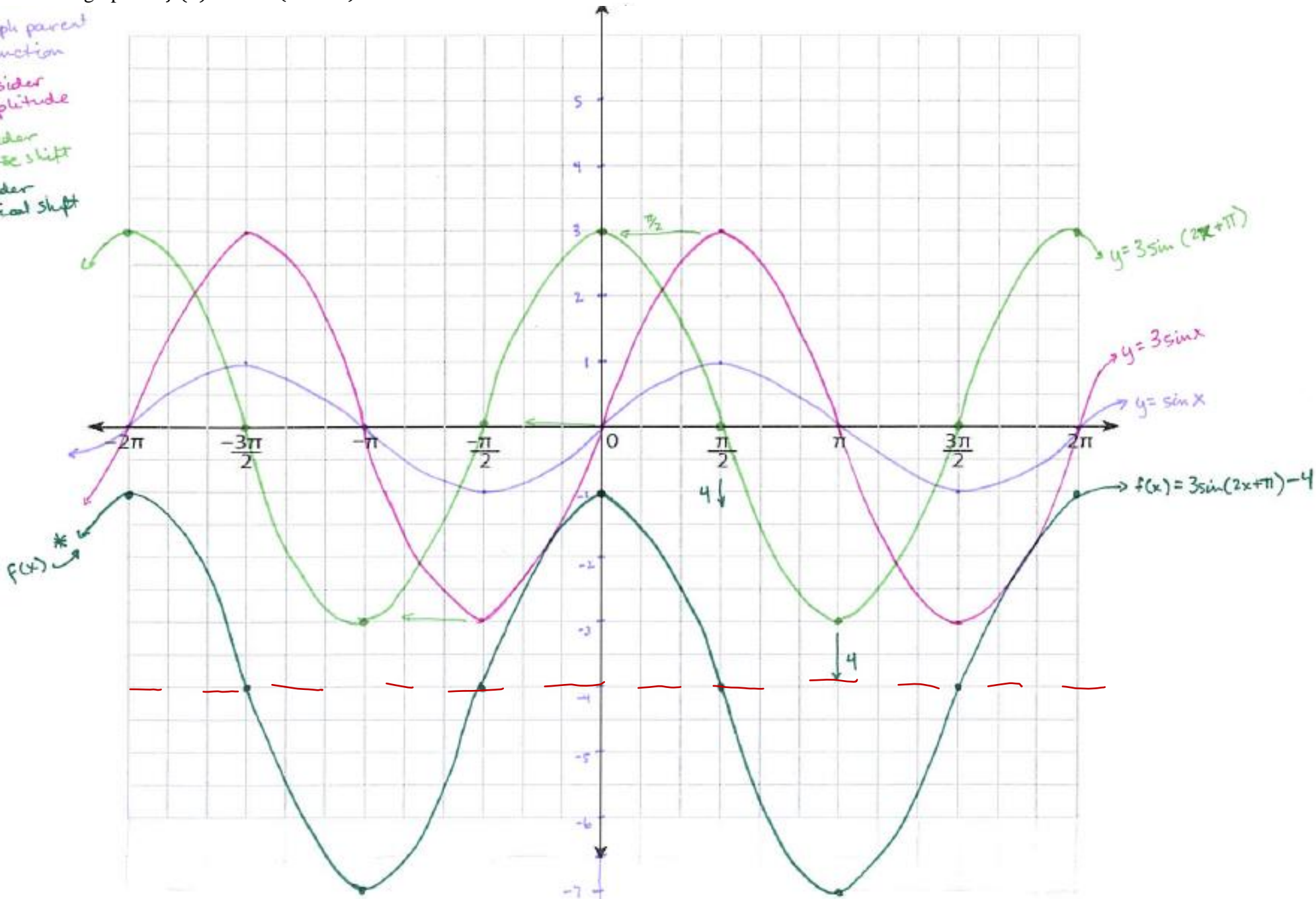
Transformation	General Form $f(x) = a \sin(bx + c) + d$ OR $f(x) = a \cos(bx + c) + d$	Example $f(x) = 3 \sin(2x + \pi) - 4$
Amplitude	$ a $	3
Period	$\frac{2\pi}{ b }$	$\frac{2\pi}{ 2 } = \frac{2\pi}{2} = \pi$
Phase Shift	$\frac{c}{b}$	$\frac{\pi}{2}$ (to the left)
Vertical Shift	d	4 (down)

Unit 5 (Chapter 4): Trigonometric Functions

DATE: _____
Pre-Calculus 2016-2017

Sketch the graph of: $f(x) = 3 \sin(2x + \pi) - 4$

- ① graph parent function
- ② consider amplitude
- ③ consider phase shift
- ④ consider vertical shift



Unit 5 (Chapter 4): Trigonometric Functions

More Practice

Transformations of Sinusoidal Functions

<https://www.khanacademy.org/math/trigonometry/trig-function-graphs>

<http://www.purplemath.com/modules/grphtrig.htm>

http://www.algebralab.org/lessons/lesson.aspx?file=Trigonometry_TrigTransformations.xml

<https://www.youtube.com/watch?v=iEbF1aa0Qps>

https://www.youtube.com/watch?v=s_NI50p-pcg

Homework Assignment

p.394 #3,5,9,11,13,15,21,25

SAT Connection

Solution

Choice C is correct. Since the angles are acute and $\sin(a^\circ) = \cos(b^\circ)$, it follows from the complementary angle property of sines and cosines that $a + b = 90$. Substituting $4k - 22$ for a and $6k - 13$ for b gives $(4k - 22) + (6k - 13) = 90$, which simplifies to $10k - 35 = 90$. Therefore, $10k = 125$, and $k = 12.5$.

Choice A is incorrect and may be the result of mistakenly assuming that $a + b$ and making a sign error. Choices B and D are incorrect because they result in values for a and b such that $\sin(a^\circ) \neq \cos(b^\circ)$.