DATE: ______________

4.4 Optimization Problems

To solve optimization problems,

① Draw a picture, ID what we know, ID what we need to find.

② Write formula(s).

③ Express the quantity to be maximized/minimize as a function of one variable.

④ Find the interval for the variable (if necessary)

⑤ Find the max/min (DERIVE!)

⑥ Write the answer

Example 1:
Find the two positive numbers whose sum of the first and twice the second is 100 and the product is a maximum.

① $x \to 1^{st}$ # $Sum = 100$ max product = ?
$y \to 2^{nd}$ # $x = ?$
$y = ?$

② $S = x + 2y$ $P = xy$

③ $100 = x + 2y$
$100 - 2y = x$
$P = xy$
$P(y) = (100 - 2y)y$
$P(y) = 100y - 2y^2$

⑤ $P'(y) = 100 - 4y$
$0 = 100 - 4y$ crit #s
$4y = 100$
$y = 25$

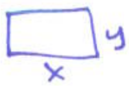
④ $x > 0 \rightarrow 100 - 2y > 0$
$y > 0$
$-2y > -100$
$y < 50$
$0 < y < 50$

$P''(y) = -4$
$P''(25) = -4$,
so $y = 25$ is a max b/c $P''(25) < 0$

$100 - 2y = x$
$100 - 2(25) = x$
$50 = x$

$\boxed{\text{The two positive \#s are 25 and 50}}$

Example 2:
Find the dimensions of a rectangle with a perimeter of 100m, whose area is as large as possible.

① 

$P = 100m$ $\qquad$ $A_{max} = ?$
$x = ?$
$y = ?$

② $P = 2x + 2y$ $\qquad$ $A = xy$

③ $100 = 2x + 2y$
$100 - 2x = 2y$
$50 - x = y$

$A = xy$
$A(x) = x(50 - x)$
$= 50x - x^2$

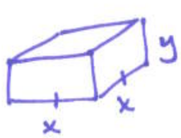
④ $0 < x < 50$
$y > 0$

⑤ $A'(x) = 50 - 2x$
$0 = 50 - 2x$
crit # → $25 = x$

$A''(x) = -2$
$A''(25) = -2$
so $x = 25$ is a max

$\boxed{\text{The dimensions that give the greatest area are } 25m \times 25m}$

Example 3:
A manufacturer needs to design an open box with a square base and a surface area of 108 in^2. What dimensions will make a box with a maximum volume?

① 

$SA = 108 in^2$ $\qquad$ $V_{max} = ?$
$x = ?$
$y = ?$

② $SA = x^2 + 4xy$ $\qquad$ $V = l \cdot w \cdot h$
$V = x^2 y$

③ $108 = x^2 + 4xy$
$108 - x^2 = 4xy$
$\frac{108 - x^2}{4x} = y$

$V = x^2 y$
$V(x) = x^2\left(\frac{108 - x^2}{4x}\right)$
$= \frac{108x^2 - x^4}{4x}$
$A(x) = 27x - \frac{1}{4}x^3$

④ $x > 0$
$y > 0$ → $\frac{108 - x^2}{4x} > 0$
$x < \sqrt{108}$

⑤ $V'(x) = 27 - \frac{3}{4}x^2$
$0 = 27 - 3/4\,x^2$ ← crit #
$3/4\,x^2 = 27$
$x^2 = 36$
$x = \pm 6$ ← only 6 b/c $x > 0$

$V''(x) = -3/2\,x$
$V''(6) = -3/2(6) < 0$
so $x = 6$ is a max

$\boxed{\text{The dimensions of the box that give a max volume are } 6in \times 6in \times 3in}$