

Optimization (continued again)

Determine the dimensions of a rectangular solid (with a square base) of maximum volume if its surface area is 150 square inches.



$$\textcircled{1} \quad \begin{array}{l} \text{SA} = 150 \text{ in}^2 \\ x = ? \\ y = ? \\ V' = ? \end{array}$$

$$\textcircled{2} \quad \text{SA} = 2x^2 + 4xy$$

$$150 = 2x^2 + 4xy$$

$$150 - 2x^2 = 4xy$$

$$\frac{150 - 2x^2}{4x} = y$$

$$\text{SA} = 150 \text{ in}^2$$

$$x = ?$$

$$y = ?$$

$$V' = ?$$

$$V = x^2 y$$

$$V = x^2 \left(\frac{150 - 2x^2}{4x} \right)$$

$$V(x) = \frac{150x^2 - 2x^4}{4x}$$

$$\textcircled{3} \quad V(x) = \frac{75}{2}x - \frac{1}{2}x^3$$

(4)

Set $y = 0$...

$$\frac{150 - 2x^2}{4x} = 0$$

$$150 - 2x^2 = 0$$

$$2x^2 = 150$$

$$x^2 = 75$$

$$x = \sqrt{75}$$

$$0 < x < \sqrt{75}$$

$$\textcircled{5} \quad V'(x) = \frac{75}{2} - \frac{3}{2}x^2$$

$$0 = \frac{75}{2} - \frac{3}{2}x^2$$

$$\frac{3}{2}x^2 = \frac{75}{2}$$

$$x^2 = 25$$

$$x = 5$$

$$y = \frac{150 - 2(5)^2}{4(5)}$$

$$= \frac{150 - 50}{20}$$

$$= 5$$

(6) The dimensions are: 5 in \times 5 in \times 5 in