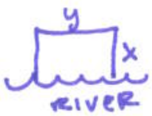
1. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 180,000 square meters in order to provide enough grass for the herd. What dimensions would require the least amount of fencing if no fencing is needed along the river?

① 

$A = 180{,}000\ m^2$

$\min P = ?$

$x = ?$

$y = ?$

② $A = xy$ $\quad P = 2x + y$

③ $180{,}000 = xy$ $\quad P = 2x + \cancel{y}$

$\frac{180000}{x} = y$ $\rightarrow$ $P(x) = 2x + \frac{180000}{x}$

④ $x > 0$

$y > 0$ $\quad \frac{180{,}000}{x} > 0$

$180{,}000 > 0$

always true, so no upper limit on y

⑤ $P'(x) = 2 - \frac{180000}{x^2}$ $\quad P''(x) = +\frac{360000}{x^3}$

$0 = 2 - \frac{180000}{x^2}$ $\quad P''(300) > 0$

so $x = 300$ is a min.

$\frac{180000}{x^2} = 2$

$2x^2 = 180000$ $\quad y = \frac{180000}{x}$

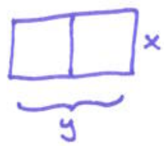
$x^2 = 90000$ $\quad y = \frac{180000}{300}$

$x = 300$

$y = 600$

The dimensions of the pasture that requires least amount of fencing are 300 m × 600m

2. A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should be used so that the enclosed area will be a maximum?

① 

$P = 200ft$

$\max A = ?$
$x = ?$
$y = ?$

② $P = 3x + 2y$ $\quad A = xy$

③ $200 = 3x + 2y$
$200 - 3x = 2y$
$100 - \frac{3}{2}x = y$

$A = xy$
$A(x) = x(100 - \frac{3}{2}x)$
$A(x) = 100x - \frac{3}{2}x^2$

④ $x > 0$
$y > 0$

$100 - \frac{3}{2}x > 0$
$100 > \frac{3}{2}x$
$\frac{200}{3} > x$

So, $0 < x < \frac{200}{3}$

⑤ $A'(x) = 100 - \frac{3}{\cancel{}}x$
$0 = 100 - \frac{3}{\cancel{}}x$
$\frac{3}{\cancel{}}x = 100$
$x = \cancel{\frac{400}{3}}\ \frac{100}{3}$

$A''(x) = -3$
$A''(\frac{100}{3}) < 0$,
So $x = \frac{100}{3}$ is a max

$y = 100 - \frac{3}{2}x$
$y = 100 - \frac{3}{2}(\frac{100}{3})$
$y = 50$

$\boxed{\text{The dimensions of the corral should be } \frac{100}{3}m \times 50m}$

3. An open box of maximum volume is to be made from a square piece of material, 24 inches on a side, by cutting equal squares from the corners and turning up the sides. Find the dimensions of the box.

① 

max $V = ?$

$x = ?$

② $V = l \cdot w \cdot h$

$V = (24-2x)(24-2x)(x)$

$V = 2(12-x) \cdot 2(12-x)(x)$

③ $V = 4(144 - 24x + x^2)(x)$

$V = 4(144x - 24x^2 + x^3)$

④ $x > 0$ $\quad 24 - 2x > 0$

$24 > 2x$

$12 > x$

so, $0 < x < 12$

⑤ $V'(x) = 4(144 - 48x + 3x^2)$

factor out a 3 → $0 = 12(x^2 - 16x + 48)$

$0 = 12(x-4)(x-12)$

$x = 4$, $\cancel{x = 12}$

↑ not in domain

$V''(x) = 4(-48 + 6x)$

$V''(4) = 4(-48 + 6(4)) < 0$

so $x = 4$ is a max

The dimensions of the box are $16\text{in} \times 16\text{in} \times 4\text{in}$

4. If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

① 

$SA = 1200 cm^2$

max $V = ?$
$x = ?$
$y = ?$

② $SA = x^2 + 4xy$ $\quad V = x^2 y$

③ $1200 = x^2 + 4yx$

$1200 - x^2 = 4xy$

$\frac{1200 - x^2}{4x} = y$

$V(x) = x^2\left(\frac{1200 - x^2}{4x}\right)$

$= \frac{1200x^2 - x^4}{4x}$

$= 300x - \frac{1}{4}x^3$

④ $x > 0$

$y > 0$

$\frac{1200 - x^2}{4x} > 0$

$1200 - x^2 > 0$

$x < \sqrt{1200}$

so, $0 < x < \sqrt{1200}$

⑤

$V'(x) = 300 - \frac{3}{4}x^2$

$0 = 300 - \frac{3}{4}x^2$

$\frac{3}{4}x^2 = 300$

$x^2 = 400$

$x = \pm 20$ ↳ only pos. 20 in domain

$x = 20$

$V''(x) = -\frac{3}{2}x$

$V''(20) < 0$

so $x = 20$ is a max

$y = \frac{1200 - x^2}{4x}$

$y = \frac{1200 - (20)^2}{4(20)}$

$y = \frac{800}{80} = 10$

$V = (20)^2(10)$

$V = 4000$

$\boxed{\text{The Largest possible volume of the box is } 4000 \text{ cm}^3}$