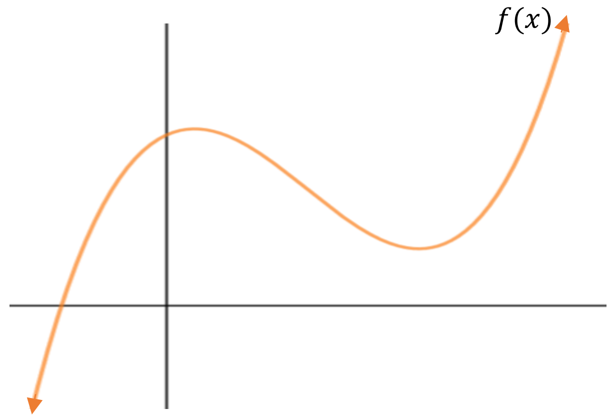
DATE: ____________________

Linearization (Linear Approximation)

<u>Linear Approximation</u>
→use of a tangent line at $(a, f(a))$ to approximate some y-value at some x-value.



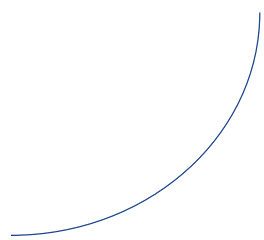
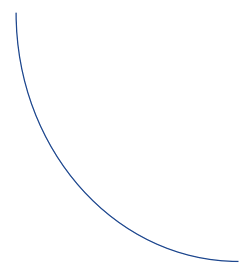
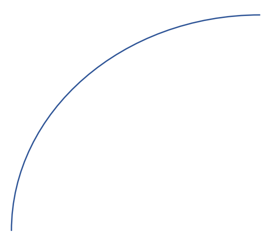
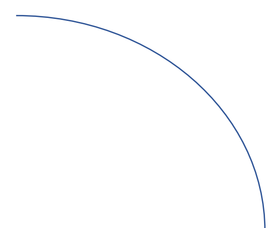
If f is diff'able at $x = a$, then the equation of the tangent line at $(a, f(a))$ is:

Example 1:
Estimate $f(4.1)$ for $f(x) = \sqrt{x^2 + 9}$.

Example 2:
The function f is twice-differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$?

Overapproximation vs. Underapproximation

Is the tangent line approximation an over or under approximation of the actual value?

	f is increasing	f is decreasing
f is concave up		
f is concave down		

Conclusion:

Tangent line approximation is an over approximation of the actual value when:

Tangent line approximation is an under approximation of the actual value when:

Example 1:

Is the tangent line approximation for $f(4.1)$ where $f(x) = \sqrt{x^2 + 9}$ an over or under approximation of the actual value of $f(4.1)$?

Example 2:

The function f is twice-differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. Is the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$ an overestimate or underestimate of $f(1.9)$?