Differentials

$\frac{dy}{dx} = f'(x)$ where $y = f(x)$ is a diff'able function

$dy \approx f'(x)\,dx$

ex: Find dy & evaluate dy for given values

$y = \frac{2x}{1+x^2}$ $\quad x = -2,\ dx = 0.1$

$$\frac{dy}{dx} = \frac{(1+x^2)(2) - (2x)(2x)}{(1+x^2)^2}$$

$$dy = \frac{2+2x^2-4x^2}{(1+x^2)^2} \cdot dx$$

$$dy = \frac{2-2x^2}{(1+x^2)^2} \cdot dx$$

$$dy = \frac{2-2(-2)^2}{(1+(-2)^2)^2} \cdot 0.1$$

$$= \frac{2-8}{25} \cdot 0.1 = \frac{-6}{25}(.1) = (-.24)(-.1)$$

$$= \boxed{-.024}$$

Ex: Estimate the increase in a sphere's volume when the radius changes from 10cm to 10.1cm

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$dV = 4\pi r^2 dr$$

$$dV = 4\pi(10)^2(.1)$$

$$= 4\pi(100)(.1)$$

$$\boxed{dV = 40\pi \text{ m}^3}$$

actual change in V (ΔV) would be

$V(10.1) - V(10)$

$\frac{4}{3}\pi(10.1)^3 - \frac{4}{3}\pi(10)^3$

$= 40.401\pi \text{ m}^3$

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