AP® CALCULUS BC FREE-RESPONSE QUESTIONS

No calculator is allowed for these problems.

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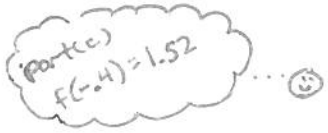
4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.



(d) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine whether the approximation found in part (c) is less than or greater than $f(-0.4)$. Explain your reasoning.

$\frac{dy}{dx} = 2x - y$

$\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx}$

$= 2 - (2x - y)$

$\frac{d^2y}{dx^2} = 2 - 2x + y$

approximation for $f(-0.4) = 1.52$ (pt. in quad II)

$\left.\frac{d^2y}{dx^2}\right|_{\text{quad II}} > 0$, $(x<0, y>0)$

Approximation in part (c) is less than $f(-0.4)$ b/c $\frac{d^2y}{dx^2} > 0$ in quad II so $f(x)$ concave up in quad II.

6. Let f be the function whose graph goes through the point (3, 6) and whose derivative is given by $f'(x) = \frac{1 + e^x}{x^2}$.

(a) Write an equation of the line tangent to the graph of f at $x = 3$ and use it to approximate $f(3.1)$.

(b) ~~Use Euler's method, starting at $x = 3$ with a step size of 0.05, to approximate $f(3.1)$.~~ Use f'' to explain why this approximation is less than $f(3.1)$.

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a) $y - y_1 = m(x - x_1)$

$y - 6 = \frac{1 + e^3}{9}(x - 3)$

$f'(3) = \frac{1 + e^3}{3^2} = \frac{1 + e^3}{9}$

$f(3.1) - 6 = \frac{1 + e^3}{9}(3.1 - 3)$

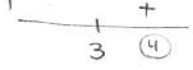
$f(3.1) = \frac{1 + e^3}{9}(.1) + 6$

b) $f''(x) = \frac{x^2(e^x) - (1 + e^x)(2x)}{(x^2)^2}$

$= \frac{x^2e^x - 2x - 2xe^x}{x^4}$

$= \frac{x(xe^x - 2 - 2e^x)}{x^4} = \frac{xe^x - 2e^x - 2}{x^3}$

$= \frac{e^x(x-2) - 2}{x^3}$



Approximation is less than $f(3.1)$ b/c $f''(x) > 0$ on $(3, \infty)$ so f concave up on $(3, \infty)$