DATE: ______________

Related Rates & Differentiating with respect to time

Differentiate with respect to t

Example 1:

$$y^2 + x^3 = 5x$$

$$2y\frac{dy}{dt} + 3x^2\frac{dx}{dt} = 5\frac{dx}{dt}$$

Example 2:

Find dy/dt when $x = 1$, $dx/dt = 2$

$$x^3 + 3 = y^2$$

$$1^3 + 3 = y^2$$
$$4 = y^2$$
$$\pm 2 = y$$

$$3x^2\frac{dx}{dt} + 0 = 2y\frac{dy}{dt}$$

$$3(1)^2(2) = 2(\pm 2)\frac{dy}{dt}$$

$$6 = \pm 4\frac{dy}{dt}$$

$$\pm\frac{3}{2} = \frac{dy}{dt}$$

Solving Related Rate Problems (6 steps)

① Make a sketch

② ID given info

③ ID what need to find

these occur at same time

④ Write equation

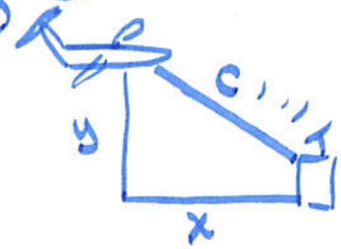
⑤ Diff'tiate equation (w/respect to time)

⑥ Sub in values & solve.

STEP 5 → DERIVE

Examples:

1. A plane flying horizontally at an altitude of 1 mi and a speed of 500 mi/hr passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 mi away from the station.

① 

② $y = 1$ mi

$\frac{dx}{dt} = 500$ mi/hr

$c = 2$ miles

③ $\frac{dc}{dt} = ?$

④ $x^2 + y^2 = c^2$

y is constant, so $\frac{dy}{dt} = 0$

⑤ $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2c\frac{dc}{dt}$

$x^2 + y^2 = c^2$

$x^2 + 1^2 = 2^2$

$x^2 = 3$

$x = \sqrt{3}$

$2(\sqrt{3})(500) + 0 = 2(2)\frac{dc}{dt}$

$1000\sqrt{3} = 4\frac{dc}{dt}$

$250\sqrt{3} = \frac{dc}{dt}$

$250\sqrt{3}$ mi/hr is rate distance from plane to station increasing

2. If a snowball melts so that its surface area decrease at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 10 cm.

$\frac{dSA}{dt} = -1$ cm^2/min

$\frac{dd}{dt} = ?$

$d = 10$cm

$SA = 4\pi r^2$

$d = 2r$

$\frac{1}{2}d = r$

$SA = 4\pi(\frac{1}{2}d)^2$

$SA = \pi d^2$

⑤ $\frac{dSA}{dt} = 2\pi d\frac{dd}{dt}$

$-1 = 2\pi(10)\frac{dd}{dt}$

$\frac{dd}{dt} = \frac{-1}{20\pi}$ cm/min

$\frac{-1}{20\pi}$ cm/min is rate diameter is decreasing

3. A ladder 25 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 3 ft/s, how fast is the ladder falling when the ladder is 7 ft away from the wall?

$l = 25\text{ ft}$

$\frac{dx}{dt} = 3\text{ ft/sec}$

$x = 7\text{ ft}$

$\frac{dy}{dt} = ?$

$x^2 + y^2 = l^2$ ⟶ Ladder is constant length (won't change size)

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(7)(3) + 2(24)\frac{dy}{dt} = 0$

$7^2 + y^2 = 25^2$

$49 + y^2 = 625$

$y^2 = 576$

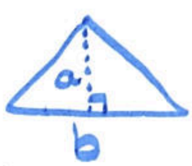
$y = 24$

$2(24)\frac{dy}{dt} = -2(7)(3)$

$\frac{dy}{dt} = \frac{-\cancel{2}(7)(\cancel{3})}{\cancel{2}(\cancel{24})8}$

$\boxed{\frac{dy}{dt} = -\frac{7}{8}\text{ ft/sec}}$

4. The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2?



$\frac{da}{dt} = 1\text{cm/min}$

$\frac{dA}{dt} = 2\text{cm}^2\text{/min}$

$a = 10\text{cm}$

$A = 100\text{cm}^2$

$\frac{db}{dt} = ?$

$A = \frac{1}{2}ba$

$\frac{dA}{dt} = \frac{1}{2}\left(a\cdot\frac{db}{dt} + b\frac{da}{dt}\right)$

$2 = \frac{1}{2}\left(10\cdot\frac{db}{dt} + 20(1)\right)$

$2 = 5\frac{db}{dt} + 10$

$-8 = 5\frac{db}{dt}$

$\boxed{\frac{db}{dt} = -\frac{8}{5}\text{ cm/min}}$

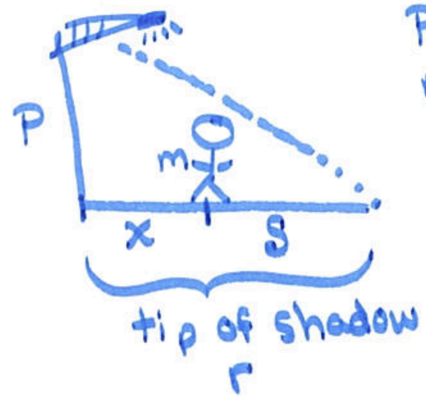
$A = \frac{1}{2}ba$

$100 = \frac{1}{2}b(10)$

$100 = 5b$

$20 = b$

5. A street light is mounted at the top of a 15-ft-tall pole. A man 6-ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?



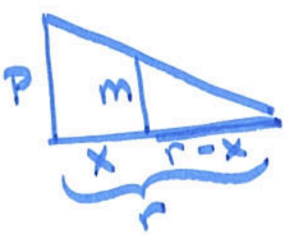
$p = 15\text{ft}$

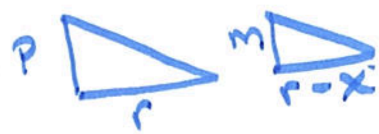
$m = 6\text{ft}$

$\frac{dx}{dt} = 5\text{ ft/sec}$

$x = 40\text{ft}$

$\frac{dr}{dt} = ?$





$$\frac{p}{m} = \frac{r}{r-x}$$

$$pr - px = mr$$

p → constant
pole won't change ~~size~~ size

m → constant
man won't change size

$$p\frac{dr}{dt} - p\frac{dx}{dt} = m\frac{dr}{dt}$$

$$15\frac{dr}{dt} - 15(5) = 6\frac{dr}{dt}$$

$$9\frac{dr}{dt} = 75 \rightarrow \frac{dr}{dt} = \frac{75}{9} = \boxed{\frac{25}{3}\ \text{ft/sec}}$$