Related Rates Problems

1. A kite is 100 ft high. There is 260 ft of string which is being reeled out at the rate of 5 ft/sec. If this results in the kite being carried along horizontally, what is the horizontal speed of the kite?

2. Helium is pumped into spherical balloon at the rate of 3π ft^3/min. At what rate is the radius increasing when the radius is 3 ft?

3. Helium is pumped into spherical balloon at the rate of 3π ft^3/min. At what rate is the surface area increasing when the radius is 3 ft?

4. A rocket is rising vertically from a point on the ground that is 100 m from an observer at ground level. The observer notes that the angle of elevation is increasing at a rate of $\pi/15$ radians/sec when the angle of elevation is $\pi/3$ radians. Find the speed of the rocket at that instant.

A kite is 100 ft high. There is 260 ft of string which is being reeled out at the rate of 5 ft/sec. If this results in the kite being carried along horizontally, what is the horizontal speed of the kite?

$h = 100\text{ft}$	$x = 100\text{ft}$
$s = 260\text{ft}$	$x = 260\text{ft}$
$h = 260\text{ft}$	$s = 100\text{ft}$
$ds/dt = 5\text{ ft/sec}$	$dh/dt = 5\text{ ft/sec}$
$dx/dt = 5\text{ ft/sec}$	$dh/dt = ?$
$dx/dt = ?$	$ds/dt = ?$
$x^2 + h^2 = s^2$	$A = \frac{1}{2}xh$

Helium is pumped into spherical balloon at the rate of 3π ft^3/min. At what rate is the radius increasing when the radius is 3 ft?

$r = 3$ ft	$V = 3$ ft
$SA = 3$ ft	$d = 3$ ft
$V = 3\pi$ ft^3/min	$r = 3\pi$ ft^3/min
$dV/dt = 3\pi$ ft^3/min	$dSA/dt = 3\pi$ ft^3/min
$dr/dt = 3\pi$ ft^3/min	dV/dt = ?
dr/dt = ?	dSA/dt = ?
$V = 4/3\ \pi r^3$	$SA = 4\pi r^2$

Helium is pumped into spherical balloon at the rate of 3π ft^3/min. At what rate is the surface area increasing when the radius is 3 ft?

$r = 3$ ft	$V = 3$ ft
$SA = 3$ ft	$d = 3$ ft
$V = 3\pi$ ft^3/min	$r = 3\pi$ ft^3/min
$dV/dt = 3\pi$ ft^3/min	$dSA/dt = 3\pi$ ft^3/min
$dr/dt = 3\pi$ ft^3/min	$dV/dt = ?$
$dr/dt = ?$	$dSA/dt = ?$
$V = 4/3\ \pi r^3$	$SA = 4\pi r^2$

A rocket is rising vertically from a point on the ground that is 100 m from an observer at ground level. The observer notes that the angle of elevation is increasing at a rate of $\pi/15$ radians/sec when the angle of elevation is $\pi/3$ radians. Find the speed of the rocket at that instant.

$y = 100$ m	$x = 100$ m
$y = \pi/15$ radians/sec	$\theta = \pi/15$ radians/sec
$\theta = \pi/3$ radians	$x = \pi/3$ radians
$d\theta/dt = \pi/15$ radians/sec	$dx/dt = \pi/15$ radians/sec
$dy/dt = \pi/15$ radians/sec	$dy/dt = ?$
$dx/dt = ?$	$d\theta/dt = ?$
$\tan\theta = y/x$	$\tan\theta = x/y$

Directions to Teacher:

- Give students related rates problems & cut out pieces of information.
- Students should match the givens, the unknown, & formula for each problem.
- Students should then solve the related rates problem.

Solutions:

1. A kite is 100 ft high. There is 260 ft of string which is being reeled out at the rate of 5 ft/sec. If this results in the kite being carried along horizontally, what is the horizontal speed of the kite?

Givens	h = 100 ft	ds/dt = 5 ft/sec	s = 260 ft
What looking for (unknown)	dx/dt = ?		
Formula	$x^2 + h^2 = s^2$		

$\frac{dx}{dt} = \frac{65}{12} ft/sec$

2. Helium is pumped into spherical balloon at the rate of 3π ft^3/min. At what rate is the radius increasing when the radius is 3 ft?

Givens	r = 3 ft	dV/dt = 3π ft^3/min
What looking for (unknown)	dr/dt = ?	
Formula	$V = 4/3\ \pi r^3$	

$\frac{dr}{dt} = \frac{1}{12} ft/min$

3. Helium is pumped into spherical balloon at the rate of 3π ft^3/min. At what rate is the surface area increasing when the radius is 3 ft?

Givens	r = 3 ft	dV/dt = 3π ft^3/min
What looking for (unknown)	dSA/dt = ?	
Formula	$V = 4/3\ \pi r^3$	$SA = 4\pi r^2$

$\frac{dSA}{dt} = 2\pi\ ft^2/min$

4. A rocket is rising vertically from a point on the ground that is 100 m from an observer at ground level. The observer notes that the angle of elevation is increasing at a rate of $\pi/15$ radians/sec when the angle of elevation is $\pi/3$ radians. Find the speed of the rocket at that instant.

Givens	x = 100 m	$\theta = \pi/3$ radians	dθ/dt = $\pi/15$ radians/sec
What looking for (unknown)	dy/dt = ?		
Formula	$\tan\theta = y/x$		

$\frac{dy}{dt} = \frac{80\pi}{3} m/sec$