DATE: _____

A cylindrical rubber cord is stretched at a constant rate of 2 cm per second. Assuming its volume does not change, how fast is its radius shrinking when its length is 100 cm and its radius is 10 cm. (Note: Volume of a cylinder is $V = \pi r^2 h$)

$\frac{dh}{dt} = 2\text{ cm/sec}$

$h = 100\text{cm}$

$r = 10\text{cm}$

$\frac{dr}{dt} = ?$

$V = \pi r^2 h$

$\frac{dV}{dt} = \pi\left[h \cdot 2r\frac{dr}{dt} + r^2\frac{dh}{dt}\right]$

$0 = \pi\left[100 \cdot 2 \cdot 10\frac{dr}{dt} + 10^2(2)\right]$

$0 = 2000\pi\frac{dr}{dt} + 200\pi$

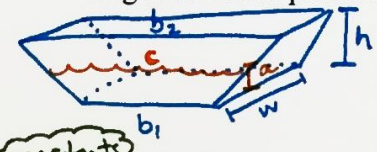
$-200\pi = 2000\pi\frac{dr}{dt}$

$-\frac{1}{10} = \frac{dr}{dt}$

Volume does not change

The radius is shrinking @ $\frac{1}{10}$ cm/sec when length is 100cm & radius is 10cm

A trapezoidal tank with a height of 8m, bottom base of 24m, top base of 36m, and width of 10m, is being filled with water. How fast does the water level rise in the tank when the water level is 4m high and water pours in at $20\text{m}^3/\text{min}$? (Note: Volume of a trapezoidal prism is $V = \frac{1}{2}(b_1 + b_2)hw$)



constants

h does not change

b_1, b_2 does not change

w does not change

$h = 8\text{m}$

$b_1 = 24\text{m}$

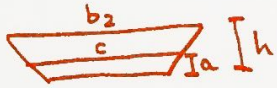
$b_2 = 36\text{m}$

$w = 10\text{m}$

$a = 4\text{m}$

$\frac{dV}{dt} = 20\text{m}^3/\text{min}$

$\frac{da}{dt} = ?$



$\frac{b_2}{h} = \frac{c}{a}$

$b_2 a = hc$

$36(4) = 8c$

$18 = c$

$b_2 a = hc$

$b_2\frac{da}{dt} = h\frac{dc}{dt}$

$36\frac{da}{dt} = 8\frac{dc}{dt}$

$\frac{9}{2}\frac{da}{dt} = \frac{dc}{dt}$

Volume Water

$V = \frac{1}{2}(b_1 + b_2)hw$

$V = \frac{1}{2}w(b_1 + c)a$

$\frac{dV}{dt} = \frac{1}{2}w\left[a\left(0 + \frac{dc}{dt}\right) + (b_1 + c)\frac{da}{dt}\right]$

$20 = \frac{1}{2}(10)\left[4\left(0 + \frac{9}{2}\frac{da}{dt}\right) + (24 + 18)\frac{da}{dt}\right]$

$20 = 5\left[18\frac{da}{dt} + 42\frac{da}{dt}\right]$

$4 = 60\frac{da}{dt}$

$\frac{1}{12} = \frac{da}{dt}$

The water level rises at $\frac{1}{12}$ m/min when water level is 4m & pours in at rate of $20\text{m}^3/\text{min}$