

4.7 Inverse Trig Functions

Target 5F: Evaluate inverse and composite trigonometric functions and expressions using the unit circle

Review of Prior Concepts

- 1) If $\sin \theta = \frac{\sqrt{2}}{2}$, find the value of θ for $-\frac{\pi}{2} \leq \theta < \frac{\pi}{2}$.
- 2) If $\cos \theta = -\frac{1}{2}$, find the value of θ for $0 \leq \theta < \pi$.
- 3) If $\tan \theta = -1$, find the value of θ for $-\frac{\pi}{2} \leq \theta < \frac{\pi}{2}$.

More Practice

Trig Values from Unit Circle

- <http://www.mathmistakes.info/facts/TrigFacts/learn/vals/sum.html>
- <http://www.kwarp.com/portfolio/trigspinner.html>
- <https://www.mathsisfun.com/geometry/unit-circle.html>
- <https://www.youtube.com/watch?v=LE6dmczMc68>
- <https://www.youtube.com/watch?v=RLjyGKWMSx0>

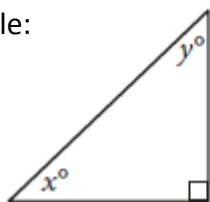


SAT Connection

Passport to Advanced Math

14. Use structure to isolate or identify a quantity of interest in an expression

Example:



In the triangle above, the sine of x° is 0.6. What is the cosine of y° ?

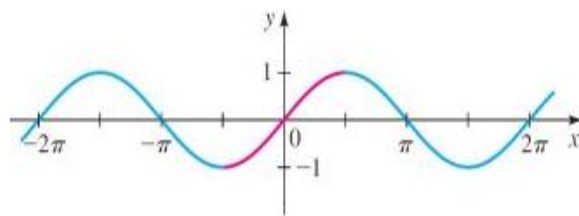
Solution

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/	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
0	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
1	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
2	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
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NOTE: You may start your answers in any column, space permitting. Columns you don't need to use should be left blank.

Inverse Sine Function

$y = \sin x$



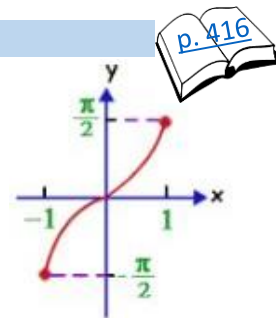
Restricted Domain: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Range: $[-1, 1]$

$y = \sin^{-1} x$

or

$y = \arcsin x$



Domain: $[-1, 1]$

Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

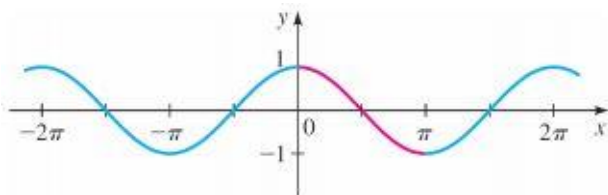
Example: Find the exact value of:

a) $\sin^{-1}(\frac{\sqrt{2}}{2})$

b) $\arcsin(-\frac{1}{2})$

Inverse Cosine Function

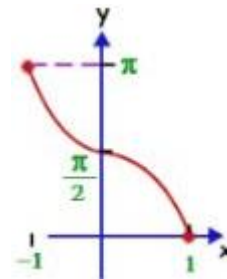
$y = \cos x$



$y = \cos^{-1} x$

or

$y = \arccos x$

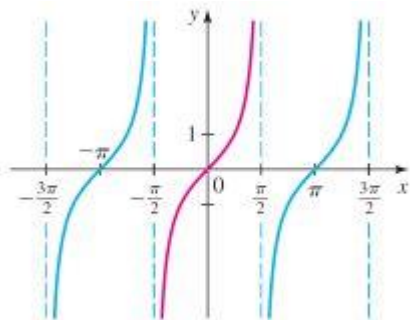
Restricted Domain: $[0, \pi]$ Range: $[-1, 1]$ Domain: $[-1, 1]$ Range: $[0, \pi]$ *Example:* Find the exact value of:

a) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

b) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$

Inverse Tangent Function

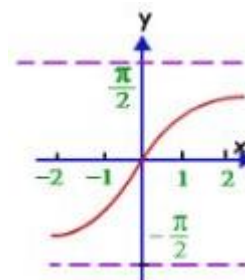
$y = \tan x$



$y = \tan^{-1} x$

or

$y = \arctan x$

Restricted Domain: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ Range: $(-\infty, \infty)$ Domain: $(-\infty, \infty)$ Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Composition of Inverse Trig Function

Composition of Inverse Trig Functions

Example:

Find the exact value of $\sin\left(\tan^{-1}\left(\frac{3}{5}\right)\right)$

① Make a Δ from inverse function

② Find missing side in Δ

③ Use Δ to get trig value of θ

Example:

Evaluate without using a calculator.

a) $\tan\left(\arccos\left(\frac{5}{7}\right)\right)$

b) $\cos(\sin^{-1} x)$

More Practice**Evaluating Inverse Trig Expressions**

<http://tutorial.math.lamar.edu/Extras/AlgebraTrigReview/InverseTrig.aspx>

<http://www.themathpage.com/atrig/inversetrig.htm>

<https://www.youtube.com/watch?v=g9S4u8eQiw>

https://www.youtube.com/watch?v=pNgkK_MR6jM

Homework Assignment

p.423 #5,9,13,19,23-29odd, 47,49,51

SAT Connection**Solution**

The correct answer is $.6$ or $\frac{3}{5}$. The angles marked x° and y° are acute angles in a right triangle. Thus, they are complementary angles. By the complementary angle relationship between sine and cosine, it follows that $\sin(x^\circ) = \cos(y^\circ)$. Therefore, the cosine of y° is $.6$. Either $.6$ or the equivalent fraction $\frac{3}{5}$ may be gridded as the correct answer.