4.7 Inverse Trig Functions

Target 5F: Evaluate inverse and composite trigonometric functions and expressions using the unit circle

Review of Prior Concepts

1) If $\sin\theta = \frac{\sqrt{2}}{2}$, find the value of θ for $-\frac{\pi}{2} \le \theta < \frac{\pi}{2}$.

$\boxed{\theta = \frac{\pi}{4}}$

2) If $\cos\theta = -\frac{1}{2}$, find the value of θ for $0 \le \theta < \pi$.

$\boxed{\theta = \frac{2\pi}{3}}$ $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

3) If $\tan\theta = -1$, find the value of θ for $-\frac{\pi}{2} \le \theta < \frac{\pi}{2}$.

$\frac{\sin\theta}{\cos\theta} = -1 = \frac{-\sqrt{2}/2}{\sqrt{2}/2}$

$\boxed{\theta = -\pi/4}$

More Practice

Trig Values from Unit Circle

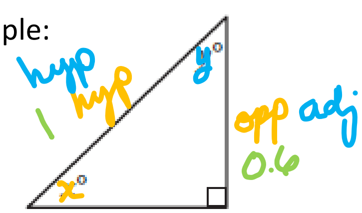
http://www.mathmistakes.info/facts/TrigFacts/learn/vals/sum.html
http://www.kwarp.com/portfolio/trigspinner.html
https://www.mathsisfun.com/geometry/unit-circle.html
https://www.youtube.com/watch?v=LE6dmczMc68
https://www.youtube.com/watch?v=RLjyGKWMSx0

SAT Connection

Passport to Advanced Math

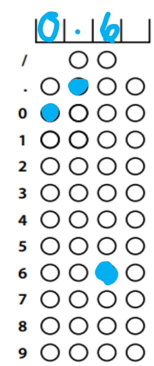
14. Use structure to isolate or identify a quantity of interest in an expression

Example:



$\cos y = \frac{0.6}{1}$

$= 0.6$



NOTE: You may start your answers in any column, space permitting. Columns you don't need to use should be left blank.

In the triangle above, the sine of $x°$ is 0.6. What is the cosine of $y°$?

Solution

Inverse Sine Function

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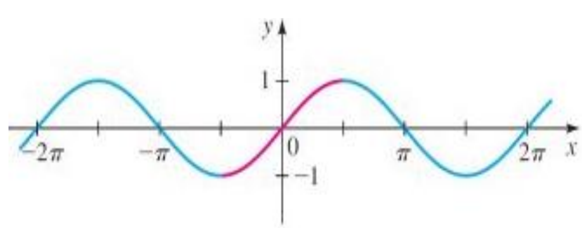
$y = \sin x$



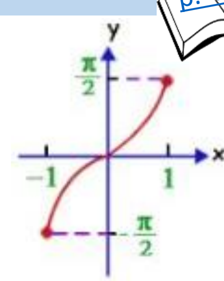
Restricted Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Range: $[-1,1]$

$y = \sin^{-1} x$

or

$y = \arcsin x$



Domain: $[-1,1]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Example: Find the exact value of:

a) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ What ∠ is $\sin\theta = \frac{\sqrt{2}}{2}$?...

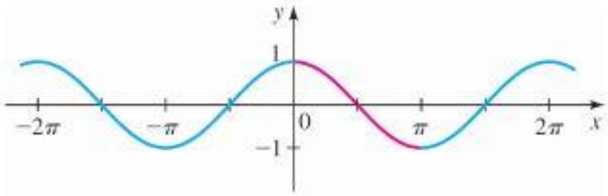
$= \frac{\pi}{4}$

b) $\arcsin\left(-\frac{1}{2}\right)$ For what ∠ is $\sin\theta = -\frac{1}{2}$...

$= -\frac{\pi}{6}$

Inverse Cosine Function

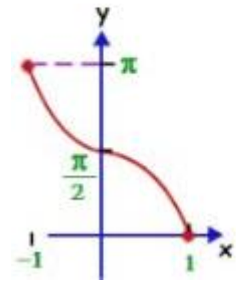
$y = \cos x$



$y = \cos^{-1} x$

or

$y = \arccos x$



Restricted Domain: $[0, \pi]$

Range: $[-1, 1]$

Domain: $[-1, 1]$

Range: $[0, \pi]$

Example: Find the exact value of:

a) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ for what ∠ is $\cos\theta = \frac{\sqrt{3}}{2}$? ... ☺

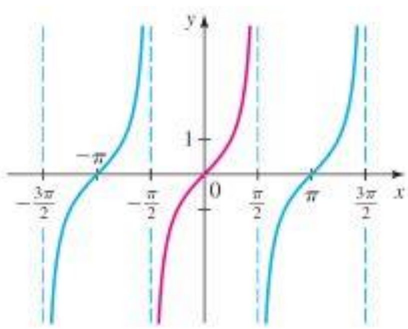
$= \frac{\pi}{6}$

b) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ for what ∠ is $\cos\theta = -\frac{\sqrt{3}}{2}$ ... ☺

$= \frac{5\pi}{6}$

Inverse Tangent Function

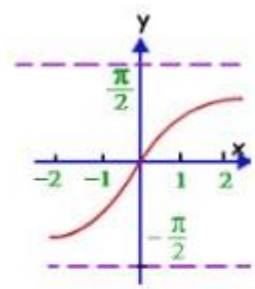
$y = \tan x$



$y = \tan^{-1} x$

or

$y = \arctan x$



Restricted Domain: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Range: $(-\infty, \infty)$

Domain: $(-\infty, \infty)$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Composition of Inverse Trig Function

Composition of Inverse Trig Functions

① Make a Δ from inverse function

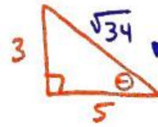
② Find missing side in Δ

③ Use Δ to get trig value of θ

Example:

Find the exact value of $\sin\left(\tan^{-1}\left(\frac{3}{5}\right)\right)$

for what ∠ is $\tan\theta = \frac{3}{5}$?

① 

② $3^2+5^2=c^2$
$9+25=c^2$
$34=c^2$
$\sqrt{34}=c$

③ $\sin(\theta)$ — opp/hyp

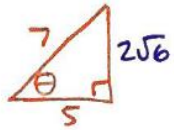
$\boxed{\frac{3}{\sqrt{34}}}$

Example:

Evaluate without using a calculator.

a) $\tan\left(\arccos\left(\frac{5}{7}\right)\right)$

① what ∠ is $\cos\theta = \frac{5}{7}$



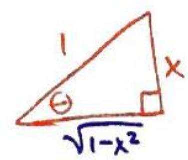
② $5^2+a^2=7^2$
$25+a^2=49$
$a^2=24$
$a=\sqrt{24}$
$a=2\sqrt{6}$

③ $\tan(\theta)$ — opp/adj

$=\boxed{\frac{2\sqrt{6}}{5}}$

b) $\cos(\sin^{-1}x)$

① what ∠ is $\sin\theta = \frac{x}{1}$



② $a^2+x^2=1^2$
$a^2+x^2=1$
$a^2=1-x^2$
$a=\sqrt{1-x^2}$

③ $\cos(\theta)$ — adj/hyp

$=\frac{\sqrt{1-x^2}}{1}$

$=\boxed{\sqrt{1-x^2}}$

More Practice

Evaluating Inverse Trig Expressions

http://tutorial.math.lamar.edu/Extras/AlgebraTrigReview/InverseTrig.aspx
http://www.themathpage.com/atrig/inversetrig.htm
https://www.youtube.com/watch?v=g9S4u8eQiww
https://www.youtube.com/watch?v=pNgkK_MR6jM

Homework Assignment

p.423 #5,9,13,19,23-29odd,47,49,51

SAT Connection
Solution

The correct answer is .6 or $\frac{3}{5}$. The angles marked $x°$ and $y°$ are acute angles in a right triangle. Thus, they are complementary angles. By the complementary angle relationship between sine and cosine, it follows that $\sin(x°) = \cos(y°)$. Therefore, the cosine of $y°$ is .6. Either .6 or the equivalent fraction $\frac{3}{5}$ may be gridded as the correct answer.