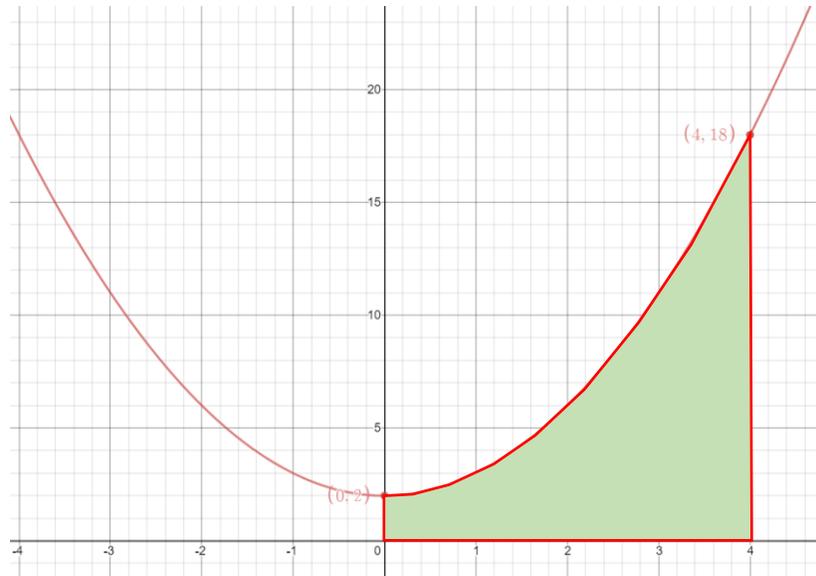


## Estimating Area with Riemann Sums

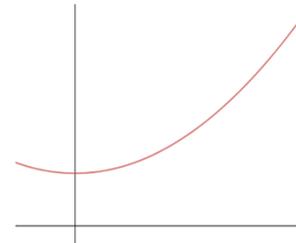
Estimate the area of the shaded region.



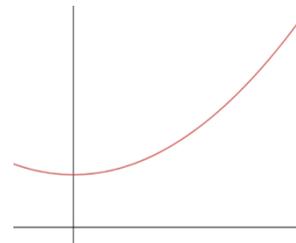
## Riemann Sums (Rectangular Approximation Method)

Georg Friedrich Bernhard Riemann (1826-1866) – German Mathematician who used rectangles to find the area of regions with linear or non-linear sides (area under a curve)

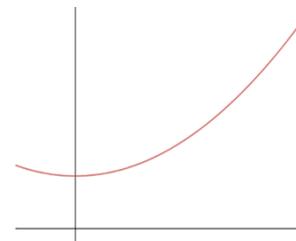
- ❖ Left Sum – use left rectangles
  - $y$ -values from the left side



- ❖ Right Sum – use right rectangles
  - $y$ -values from the right side

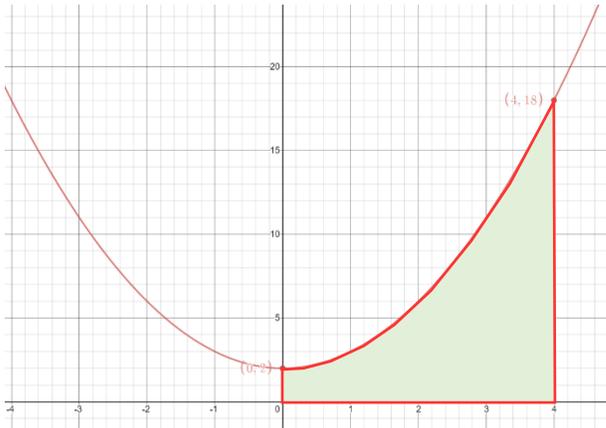


- ❖ Midpoint Sum – use midpoint rectangles
  - $y$ -values from the midpoints



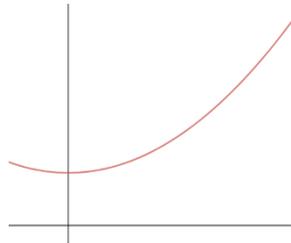
*Example:*

Find left and right sums for the region bounded by  $y = x^2 + 2$  and  $x$ -axis between  $x = 0$  and  $x = 4$  using 4 equal subintervals.



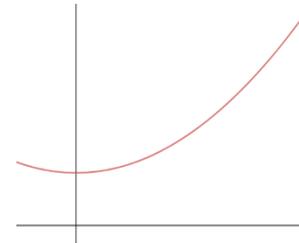
*Notice:*

This left sum



is an underapproximation  
of the actual area

This right sum



is an overapproximation  
of the actual area

so,  $\text{left sum} < \text{actual area} < \text{right sum}$

## Overestimate vs. Underestimate

### Left Riemann Sum (Left Rectangles)

	$f$ increasing	$f$ decreasing
$f$ concave up		
$f$ concave down		

Conclusion:

### Right Riemann Sum (Right Rectangles)

	$f$ increasing	$f$ decreasing
$f$ concave up		
$f$ concave down		

Conclusion: