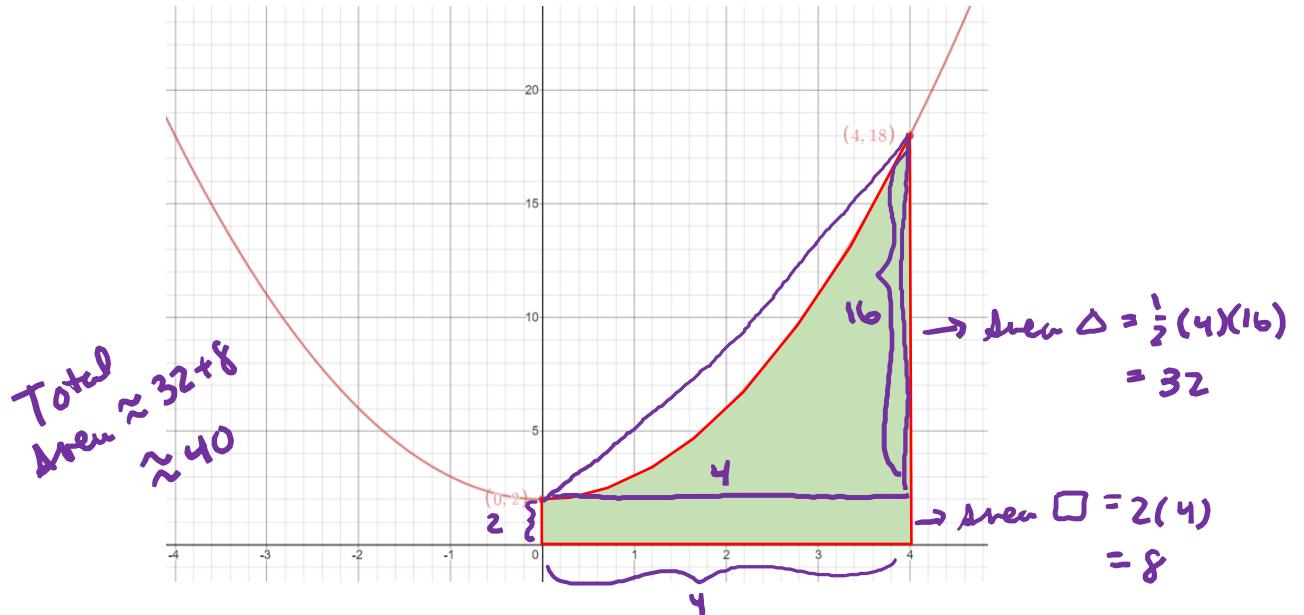


DATE: _____

Estimating Area with Riemann Sums

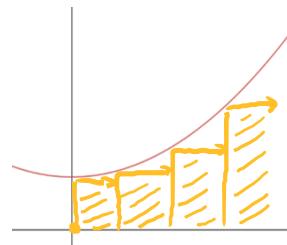
Estimate the area of the shaded region.



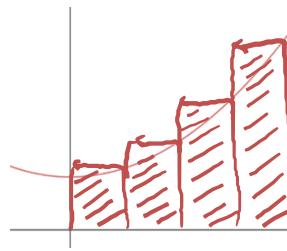
Riemann Sums (Rectangular Approximation Method)

Georg Friedrich Bernhard Riemann (1826-1866) – German Mathematician who used rectangles to find the area of regions with linear or non-linear sides (area under a curve)

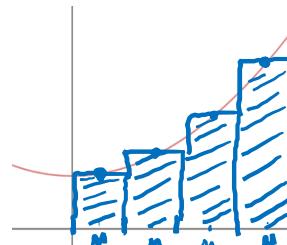
- ❖ Left Sum – use left rectangles
 - y -values from the left side



- ❖ Right Sum – use right rectangles
 - y -values from the right side

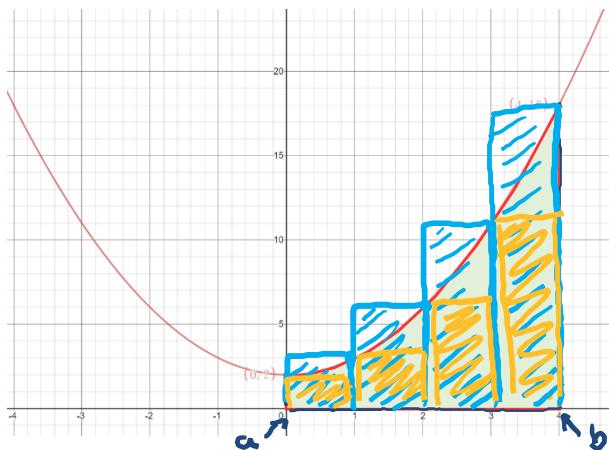


- ❖ Midpoint Sum – use midpoint rectangles
 - y -values from the midpoints



Example:

Find left and right sums for the region bounded by $y = x^2 + 2$ and x -axis between $x = 0$ and $x = 4$ using 4 subintervals.



$\Delta x \rightarrow \text{width of rectangles}$

$$\Delta x = \frac{b-a}{n} \quad \text{where } (a, b)$$

→ # of rectangles/
Subintervals

$$\Delta x = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
$f(x)$	2	3	6	11	18

Left sum:

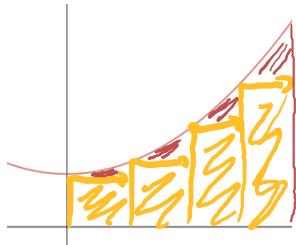
$$\begin{aligned}\text{Area} &\approx \Delta x \cdot f(0) + \Delta x \cdot f(1) + \Delta x \cdot f(2) + \Delta x \cdot f(3) \\ &\approx 1 \cdot 2 + 1 \cdot 3 + 1 \cdot 6 + 1 \cdot 11 \\ &\approx 22\end{aligned}$$

right sum:

$$\begin{aligned}\text{Area} &\approx \Delta x \cdot f(4) + \Delta x \cdot f(3) + \Delta x \cdot f(2) + \Delta x \cdot f(1) \\ &\approx 1 \cdot 18 + 1 \cdot 11 + 1 \cdot 6 + 1 \cdot 3 \\ &\approx 38\end{aligned}$$

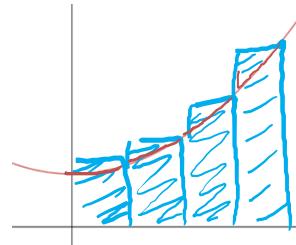
Notice:

This left sum



is an underapproximation of the actual area

This right sum



is an overapproximation of the actual area

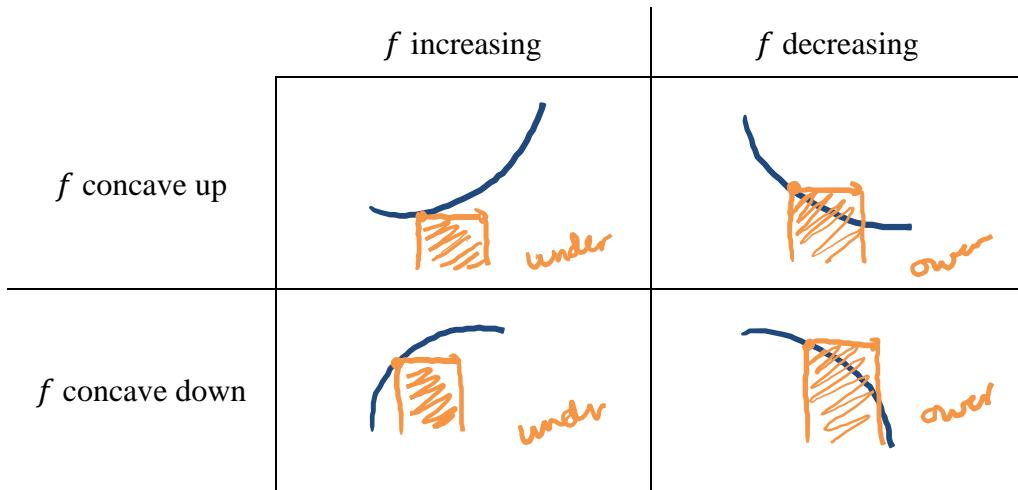
so,

$$22 < \text{actual area} < 38$$

∴ our estimation by Riemann standards was too big ... (1)

Overestimate vs. Underestimate

Left Riemann Sum (Left Rectangles)

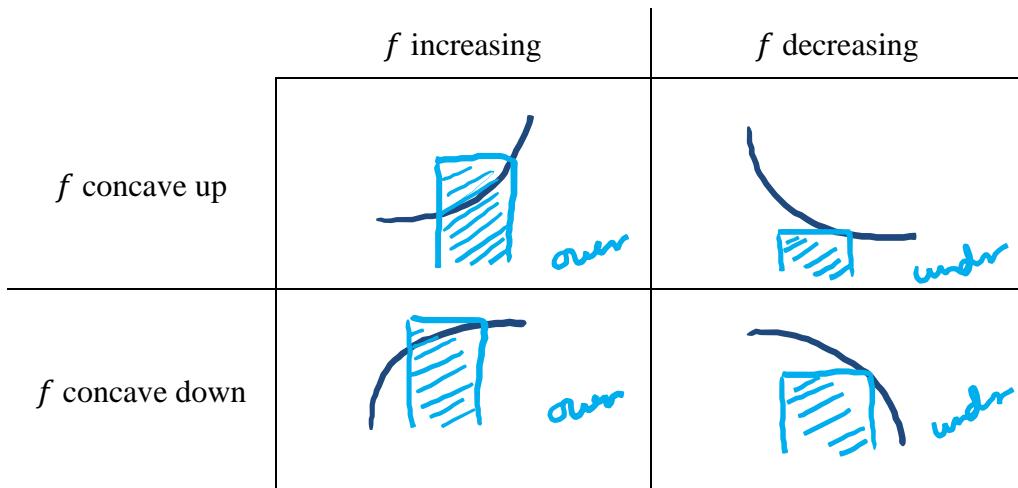


Conclusion:

when $f(x)$ inc, left sum underestimates the actual area.

when $f(x)$ dec, left sum overestimates the actual area.

Right Riemann Sum (Right Rectangles)



Conclusion:

when $f(x)$ inc, right sum overestimates actual area

when $f(x)$ dec, right sum underestimates actual area.