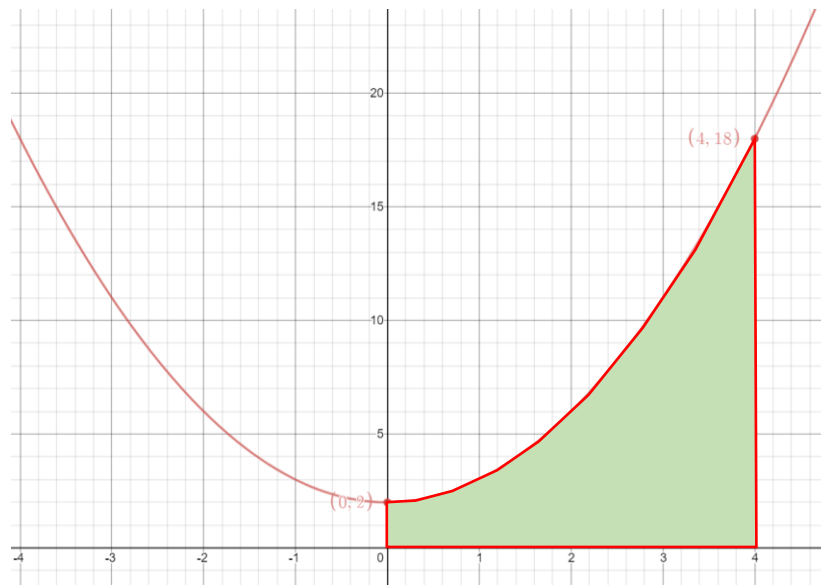


Estimating Area with Riemann Sums

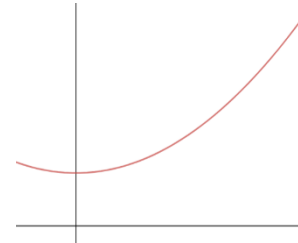
Estimate the area of the shaded region.



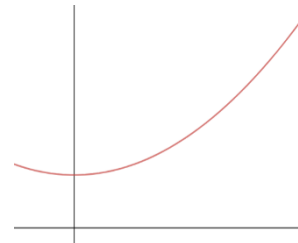
Riemann Sums (Rectangular Approximation Method)

Georg Friedrich Bernhard Riemann (1826-1866) – German Mathematician who used rectangles to find the area of regions with linear or non-linear sides (area under a curve)

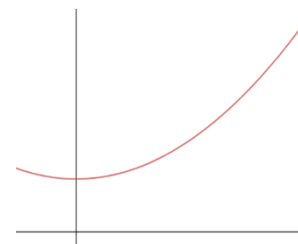
- ❖ Left Sum – use left rectangles
 - y -values from the left side



- ❖ Right Sum – use right rectangles
 - y -values from the right side



- ❖ Midpoint Sum – use midpoint rectangles
 - y -values from the midpoints



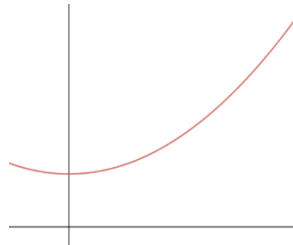
Example:

Find left and right sums for the region bounded by $y = x^2 + 2$ and x -axis between $x = 0$ and $x = 4$ using 4 subintervals.



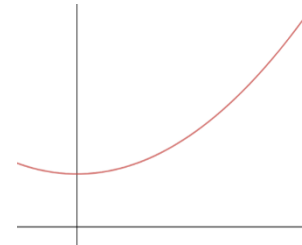
Notice:

This left sum



is an underapproximation
of the actual area

This right sum



is an overapproximation
of the actual area

so,

$L_4 < \text{actual area} < R_4$

Overestimate vs. Underestimate

Left Riemann Sum (Left Rectangles)

	f increasing	f decreasing
f concave up		
f concave down		

Conclusion:

Right Riemann Sum (Right Rectangles)

	f increasing	f decreasing
f concave up		
f concave down		

Conclusion: