DATE: _____

Unit 6 (Chapter 5): Analytic Trigonometry **Pre-Calculus 2016-2017**

5.1 Fundamental Trig Identities

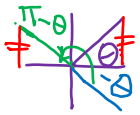
Target 6A: Verify, evaluate, and apply trigonometric identities and formulas

Review of Prior Concepts

If $\sin\theta = 0.57$, then $\sin(\pi - \theta) = ?$

$\theta = \sin^{-1}(.57)$ $\quad \sin(\pi - 0.594) = 0.57$

$\theta = 0.594$



$\sin\theta = \sin(\pi - \theta)$

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Examples

Simplify.

a) $\cot x \tan x$

$= \frac{1}{\tan x} \cdot \tan x$

$= \boxed{1}$

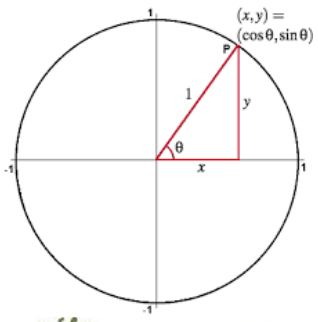
b) $\frac{\sin b}{\tan b}$

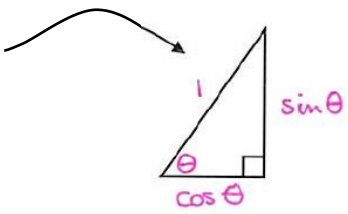
$= \frac{\sin b}{\frac{\sin b}{\cos b}}$

$= \sin b \cdot \frac{\cos b}{\sin b}$

$= \boxed{\cos b}$

Pythagorean Identities





$a^2 + b^2 = c^2$

$(\cos\theta)^2 + (\sin\theta)^2 = 1^2$

$\boxed{\cos^2\theta + \sin^2\theta = 1}$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

use, $\cos^2\theta + \sin^2\theta = 1$

divide by $\cos^2\theta$

$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$1 + \left(\frac{\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1}{\cos\theta}\right)^2$

$\boxed{1 + \tan^2\theta = \sec^2\theta}$

$\tan^2\theta = \sec^2\theta - 1$

$1 = \sec^2\theta - \tan^2\theta$

use, $\cos^2\theta + \sin^2\theta = 1$

divide by $\sin^2\theta$

$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$\left(\frac{\cos\theta}{\sin\theta}\right)^2 + 1 = \left(\frac{1}{\sin\theta}\right)^2$

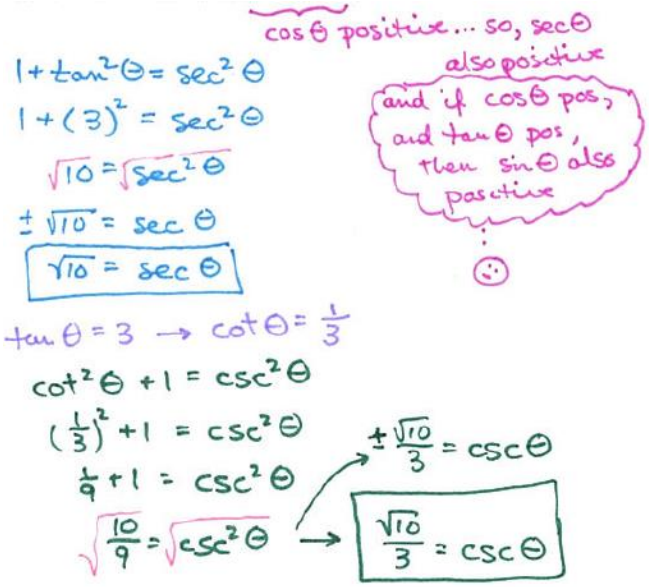
$\boxed{\cot^2\theta + 1 = \csc^2\theta}$

$\cot^2\theta = \csc^2\theta - 1$

$1 = \csc^2\theta - \cot^2\theta$

Examples

a) Use Pythagorean Identities to find $\sec\theta$ and $\csc\theta$ if $\tan\theta = 3$ and $\cos\theta > 0$.



b) Simplify $\dfrac{\sin^2\alpha+\tan^2\alpha+\cos^2\alpha}{\sec\alpha}$

$$= \frac{\sin^2\alpha + \cos^2\alpha + \tan^2\alpha}{\sec\alpha}$$

$$= \frac{1 + \tan^2\alpha}{\sec\alpha}$$

$$= \frac{\sec^2\alpha}{\sec\alpha}$$

$$= \boxed{\sec\alpha}$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$$

$$\sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta \qquad \csc\left(\frac{\pi}{2}-\theta\right) = \sec\theta$$

Odd-Even Identities

$$\sin(-x) = -\sin x \qquad \csc(-x) = -\csc x$$

$$\cos(-x) = \cos x \qquad \sec(-x) = \sec x$$

$$\tan(-x) = -\tan x \qquad \cot(-x) = -\cot x$$

Examples

a) If $\tan\left(\frac{\pi}{2}-\theta\right) = -5.326$, find $\cot\theta$.

$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$

$\therefore$, $\boxed{\cot\theta = -5.326}$

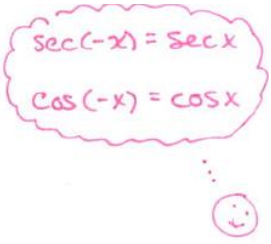
b) Simplify $\sec(-x)\cos(-x)$

$\sec(-x)\cos(-x)$

$= \sec x \cdot \cos x$

$= \frac{1}{\cos x} \cdot \cos x$

$= \boxed{1}$



More Practice

Fundamental Trig Identities

http://www.intmath.com/analytic-trigonometry/1-trigonometric-identities.php
http://www.mathguide.com/lessons2/TrigExpress.html
http://www.purplemath.com/modules/proving.htm
https://www.youtube.com/watch?v=CsfHFZL345M
https://www.youtube.com/watch?v=I4mcja8abDc

Homework Assignment

p.451 #3,5,9,13-23odd