

Readings of the power being used at a given time are taken at hourly intervals at an electric substation. The units of power are in kilowatts (kW) and the hourly readings over a 12-hour period are shown below.

$t$ (hours)	0	1	2	3	4	5	6	7	8	9	10	11	12
$f(t)$ (kW)	900	1200	1300	1200	1000	850	1200	1000	1100	1400	1400	1700	1500

$\Delta x = 2$        $\Delta x = 2$        $\Delta x = 2$        $\Delta x = 2$

$\Delta x = \frac{b-a}{n} = \frac{12-4}{4} = \frac{8}{4} = 2$

- a) Using a left sum approximation with 4 equally spaced subintervals, approximate the power usage from time  $t = 4$  to  $t = 12$ .

$$\begin{aligned}
 \text{left sum} &= 2(1000) + 2(1200) + 2(1100) + 2(1400) \\
 &= 9400 \text{ hr(kW)} \\
 &= 9400 \text{ kWh}
 \end{aligned}$$

- b) Using a right sum approximation with 4 equally spaced subintervals, approximate the power usage from time  $t = 4$  to  $t = 12$ .

$$\begin{aligned}
 \text{right sum} &= 2(1500) + 2(1400) + 2(1100) + 2(1200) \\
 &= 10400 \text{ kWh}
 \end{aligned}$$

- c) Using a midpoint approximation with 4 equally spaced subintervals, approximate the power usage from time  $t = 4$  to  $t = 12$ .

$$\begin{aligned}
 \text{midpoint sum} &= 2(850) + 2(1000) + 2(1400) + 2(1700) \\
 &= 9900 \text{ kWh}
 \end{aligned}$$

## AP Riemann Problems

$$\Delta x = \frac{9-1}{4} = \frac{8}{4} = 2$$

$x$	1	3	5	7	9
$h(x)$	2	3	3	4	5

1. Using the table of values shown above for the continuous function  $h(x)$ , which of the following is the approximation of the area under the curve  $h(x)$  using left sum with 4 equal subintervals?

(A) 34      (B) 30      (C) 28      (D) 27      (E) 24

$$\begin{aligned} \text{left sum} &= 2(2) + 2(3) + 2(3) + 2(4) \\ &= 4 + 6 + 6 + 8 \\ &= 24 \end{aligned}$$

2. Using the table of values shown above for the continuous function  $h(x)$ , which of the following is the approximation of the area under the curve  $h(x)$  using right sum with 4 equal subintervals?

(A) 34      (B) 30      (C) 28      (D) 27      (E) 24

$$\Delta x = \frac{9-1}{4} = \frac{8}{4} = 2$$

$$\begin{aligned} \text{right sum} &= 2(5) + 2(4) + 2(3) + 2(3) \\ &= 10 + 8 + 6 + 6 = 30 \end{aligned}$$

3. Using the table of values shown above for the continuous function  $h(x)$ , which of the following is the approximation of the area under the curve  $h(x)$  using midpoint sum with 2 equal subintervals?

(A) 34      (B) 30      (C) 28      (D) 27      (E) 24

$$\begin{aligned} \text{midpoint sum} &= 4(3) + 4(4) \\ &= 12 + 16 = 28 \end{aligned}$$

$$\Delta x = \frac{9-1}{2} = \frac{8}{2} = 4$$

4. A function  $f$  is continuous on  $[1,5]$  and some of the values of  $f$  are shown below:

$x$	1	4	5
$f(x)$	7	$b$	2

not equal intervals.

If the right Riemann sum is 17, then the value of  $b$  is:

(A) -4      (B) 0      (C) 3      (D) 4.25      (E) 5

$$\begin{aligned} \text{right sum} &= \Delta x \cdot f(5) + \Delta x \cdot f(4) \\ 17 &= 1 \cdot 2 + 3 \cdot b \\ 17 &= 2 + 3b \\ 15 &= 3b \\ 5 &= b \end{aligned}$$