5.1 Fundamental Trig Identities

Target 6A: Verify, evaluate, and apply trigonometric identities and formulas

Review of Prior Concepts

Simplify.

a) $\frac{3x+6}{x^2+5x+6} = \frac{3(\cancel{x+2})}{(x+3)(\cancel{x+2})}$

$= \frac{3}{x+3}$

factor + reduce

b) $\frac{x}{2}+\frac{2}{x}-\frac{x}{4} = \frac{2x \cdot x}{2x \cdot 2}+\frac{4 \cdot 2}{4 \cdot x}-\frac{x \cdot x}{4 \cdot x}$

$= \frac{2x^2+8-x^2}{4x}$

$= \frac{x^2+8}{4x}$

common denominator

More Practice

Simplifying Algebraic Expressions

http://www.purplemath.com/modules/rtnldefs3.htm
https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/simplify-rational-expressions/v/simplifying-rational-expressions-1
http://www.mathwarehouse.com/algebra/rational-expression/how-to-simplify-rational-expressions.php
https://www.youtube.com/watch?v=5xgRFbkQYgE
https://www.youtube.com/watch?v=NjBvycGQX48

Simplify the following trig expressions. Check your answer in your graphing calculator.

As a class,

1) $\tan^2 x\,(\csc^2 x - 1)$

$= \tan^2 x\,(1+\cot^2 x - 1)$

$\csc^2 x = 1+\cot^2 x$

$= \tan^2 x\,(\cot^2 x)$

$= \tan^2 x\left(\frac{1}{\tan^2 x}\right)$

$= \boxed{1}$

2) $\frac{\tan x}{\csc^2 x}+\frac{\tan x}{\sec^2 x}$

$= \frac{\tan x}{\frac{1}{\sin^2 x}} + \frac{\tan x}{\frac{1}{\cos^2 x}}$

$\csc^2 x = \frac{1}{\sin^2 x}$

$\sec^2 x = \frac{1}{\cos^2 x}$

$= \tan x \cdot \sin^2 x + \tan x \cdot \cos^2 x$

$\sin^2 x + \cos^2 x = 1$

$= \tan x\,(\sin^2 x + \cos^2 x)$

$= \tan x\,(1)$

$= \boxed{\tan x}$

Now, you try:

1) $\frac{\cos^2 x}{1-\cos^2 x}$

$= \frac{\cos^2 x}{\sin^2 x + \cos^2 x - \cos^2 x}$

$\sin^2 x + \cos^2 x = 1$

$= \frac{\cos^2 x}{\sin^2 x}$

$= \boxed{\cot^2 x}$

2) $\frac{\sin x}{\cot^2 x}+\frac{\sin x}{\cos^2 x}$

$= \frac{\sin x}{\frac{1}{\tan^2 x}} + \frac{\sin x}{\frac{1}{\sec^2 x}}$

$\cot^2 x = \frac{1}{\tan^2 x}$

$\cos^2 x = \frac{1}{\sec^2 x}$

$\tan^2 x + 1 = \sec^2 x$

$= \sin x \cdot \tan^2 x + \sin x \cdot \sec^2 x$

$= \sin x\,(\tan^2 x + \sec^2 x)$

$= \sin x\,(\tan^2 x + \tan^2 x + 1)$

$= \boxed{\sin x\,(2\tan^2 x + 1)}$

Factor the expression as a single trigonometric function

As a class,

3) $\cos x - 2\sin^2 x + 1$

$= \cos x - 2(1-\cos^2 x) + 1$

$= \cos x - 2 + 2\cos^2 x + 1$

$= 2\cos^2 x + \cos x - 1$ let $u = \cos x$

$2u^2 + u - 1$

$(2u-1)(u+1)$

$= \boxed{(2\cos x - 1)(\cos x + 1)}$

Now, you try:

3) $\sin^2 x + \frac{2}{\csc x} + 1$

$= \sin^2 x + 2\sin x + 1$ let $u = \sin x$

$u^2 + 2u + 1$

$(u+1)(u+1)$

$= (\sin x + 1)(\sin x + 1)$

$= \boxed{(\sin x + 1)^2}$

ROW by ROW

Simplify each expression. Check your answer graphically.

A	B
$\frac{1-\cos^2\theta}{\sin\theta} = \frac{\sin^2\theta}{\sin\theta}$ $= \boxed{\sin\theta}$	$\frac{\sin^2\theta\cot^2\theta}{1-\sin^2\theta} = \frac{\cancel{\sin^2\theta}\cdot\frac{\cos^2\theta}{\cancel{\sin^2\theta}}}{\cos^2\theta}$ $= \frac{\cos^2\theta}{\cos^2\theta}$ $= \boxed{1}$
$\cos x\csc x\,(\sec^2 x - 1)$ $= \cos x\cdot\frac{1}{\sin x}(\tan^2 x)$ $= \frac{\cos x}{\sin x}\cdot\left(\frac{\sin^2 x}{\cos^2 x}\right)$ $= \frac{\sin x}{\cos x}$ $= \boxed{\tan x}$	$\frac{\sec^2 x - 1}{\tan x} = \frac{\tan^2 x}{\tan x}$ $= \boxed{\tan x}$

$\frac{\tan\beta + \cot\beta}{\cot\beta} = \frac{\tan\beta}{\cot\beta} + \frac{\cot\beta}{\cot\beta}$ $= \frac{\tan\beta}{\frac{1}{\tan\beta}} + 1$ $= \tan\beta \cdot \tan\beta + 1$ $= \tan^2\beta + 1$ $= \boxed{\sec^2\beta}$	$\cos\beta\,(\sec\beta - \cos\beta)$ $= \cos\beta\sec\beta - \cos^2\beta$ $= \cos\beta \cdot \frac{1}{\cos\beta} - \cos^2\beta$ $= 1 - \cos^2\beta$ $= \boxed{\sin^2\beta}$
$\frac{\tan\alpha - \tan\alpha\sin^2\alpha}{2\sin\alpha\cos\alpha}$ $= \frac{\tan\alpha(1 - \sin^2\alpha)}{2\sin\alpha\cos\alpha}$ $= \frac{\tan\alpha(\cos^2\alpha)}{2\sin\alpha\cos\alpha}$ $= \frac{\tan\alpha \cdot \cos\alpha}{2\sin\alpha}$ $= \frac{\frac{\sin\alpha}{\cancel{\cos\alpha}} \cdot \cancel{\cos\alpha}}{2\sin\alpha}$ $\frac{\sin\alpha}{2\sin\alpha}$ $= \boxed{\frac{1}{2}}$	$\frac{\sec\alpha - \cos\alpha}{2\tan\alpha\sin\alpha}$ $= \frac{\frac{1}{\cos\alpha} - \cos\alpha}{2\frac{\sin\alpha}{\cos\alpha} \cdot \sin\alpha}$ $= \frac{\frac{1-\cos^2\alpha}{\cos\alpha}}{2\frac{\sin^2\alpha}{\cos\alpha}}$ $= \frac{\frac{\sin^2\alpha}{\cos\alpha}}{2\frac{\sin^2\alpha}{\cos\alpha}}$ $= \frac{\sin^2\alpha}{\cos\alpha} \cdot \frac{\cos\alpha}{2\sin^2\alpha}$ $= \boxed{\frac{1}{2}}$

More Practice

Simplifying Trig Expressions

http://www.intmath.com/analytic-trigonometry/1-trigonometric-identities.php
http://www.mathguide.com/lessons2/TrigExpress.html
http://www.purplemath.com/modules/proving.htm
https://www.youtube.com/watch?v=CsfHFZL345M
https://www.youtube.com/watch?v=I4mcja8abDc

Homework Assignment

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