DATE: _____________

Riemann Sums as Over/Underestimates of Area

1. The rate of temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function $W'(t)$, where $W'(t)$ is measured in degrees Fahrenheit per minute and t is measure in minutes. For $0 \le t \le 20$, use a left Riemann sum with the four subintervals indicated by the data in the table to approximate the temperature of the water over these 20 minutes. Does this approximation overestimate or underestimate the actual temperature of the water?

t (minutes)	0	4	9	15	20
$W'(t)$ (°F/min)	12.1	9.7	6.1	3.1	2.3

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2. The rate R at which a solar panel delivers electricity is a differentiable function of time t. The table below shows a sample of these rates, which can be modeled as a strictly increasing function on $4 \le t \le 16$, over an 18-hour period. Use a right Riemann sum with 6 equal subdivisions to approximate the number of amps delivered by the panel from $t = 4$ to $t = 16$. Is this approximation an overestimate or underestimate of the actual number of amps?

t (hours)	4	6	8	10	12	14	16	18	20	22
$R(t)$ (amps/hour)	36	78	160	240	320	350	360	320	240	160

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3. Suppose the graph of f is decreasing on $a \le x \le b$. Then, using the same number of subdivisions, and with L, R, and M denoting, respectively, left, right and midpoint Riemann sums, it follows that:

(A) R ≤ M ≤ L **(B)** R ≤ L ≤ M **(C)** L ≤ M ≤ R **(D)** L ≤ R ≤ M **(E)** none of these