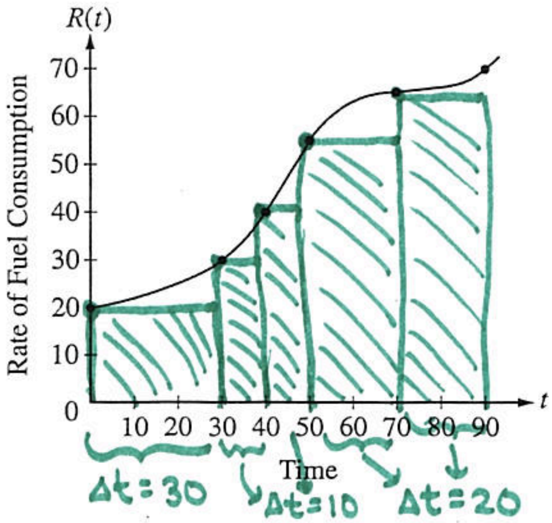


| $t$ (minutes) | $R(t)$ (gallons per minute) |
|---|---|
| 0 | 20 |
| 30 | 30 |
| 40 | 40 |
| 50 | 55 |
| 70 | 65 |
| 90 | 70 |

Approximate the area of $R(t)$ from $t = 0$ to $t = 90$ using left sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the actual area of R($t$) from $t = 0$ to $t = 90$? Explain your reasoning.

left sum $= 30(20) + 10(30) + 10(40) + 20(55) + 20(65)$

$= 3700 \text{ min}(\text{gallons}/\text{min})$

$= 3700$ gallons

Approximation is less than actual area of $R(t)$ b/c $R(t)$ is increasing.

DATE: ______________

**Riemann Sums as Over/Underestimates of Area**

**1.** The rate of temperature of water in a tub at time $t$ is modeled by a strictly ~~increasing~~ decreasing, twice-differentiable function $W'(t)$, where $W'(t)$ is measured in degrees Fahrenheit per minute and $t$ is measure in minutes. For $0 \le t \le 20$, use a left Riemann sum with the four subintervals indicated by the data in the table to approximate the temperature of the water over these 20 minutes. Does this approximation overestimate or underestimate the actual temperature of the water?

$\Delta t = 4$ $\Delta t = 5$ $\Delta t = 6$ $\Delta t = 5$

| $t$ (minutes) | 0 | 4 | 9 | 15 | 20 |
|---|---|---|---|---|---|
| $W'(t)$ (°F/min) | 12.1 | 9.7 | 6.1 | 3.1 | 2.3 |

left sum $= 4(12.1) + 5(9.7) + 6(6.1) + 5(3.1)$
$= 149$ (min)(°F/min)
$= 149$ °F

Left Riemann Sum approx is an overestimate of actual temp of water b/c $W'(t)$ is decreasing.

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**2.** The rate $R$ at which a solar panel delivers electricity is a differentiable function of time $t$. The table below shows a sample of these rates, which can be modeled as a strictly increasing function on $4 \le t \le 16$, over an 18-hour period. Use a right Riemann sum with 6 equal subdivisions to approximate the number of amps delivered by the panel from $t = 4$ to $t = 16$. Is this approximation an overestimate or underestimate of the actual number of amps?

| $t$ (hours) | 4 | 6 | 8 | 10 | 12 | 14 | 16 | ~~18~~ | ~~20~~ | ~~22~~ |
|---|---|---|---|---|---|---|---|---|---|---|
| $R(t)$ (amps/hour) | 36 | 78 | 160 | 240 | 320 | 350 | 360 | ~~320~~ | ~~240~~ | ~~160~~ |

right sum $= 2(360) + 2(350) + 2(320) + 2(240) + 2(160) + 2(78)$
$= 3016$ (hr)(amps/hr)
$= 3016$ amps

Right Riemann approx is an overestimate of actual # of amps b/c $R(t)$ is increasing on $[4, 16]$

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**3.** Suppose the graph of $f$ is decreasing on $a \le x \le b$. Then, using the same number of subdivisions, and with L, R, and M denoting, respectively, left, right and midpoint Riemann sums, it follows that:

**(A)** R ≤ M ≤ L **(B)** R ≤ L ≤ M **(C)** L ≤ M ≤ R **(D)** L ≤ R ≤ M **(E)** none of these


left sum → overestimate when f dec

right sum → underestimate when f dec

midpt sum → some over + under

R ≤ M ≤ L