

**Riemann Sums M/C & FRQ Practice**

|                         |     |     |     |     |
|-------------------------|-----|-----|-----|-----|
| $t$ (hours)             | 4   | 7   | 12  | 15  |
| $R(t)$<br>(liters/hour) | 6.5 | 6.2 | 5.9 | 5.6 |

*Handwritten notes above table:  $\Delta t=3$  (between 4 and 7),  $\Delta t=5$  (between 7 and 12),  $\Delta t=3$  (between 12 and 15). Arrows point from these intervals to the corresponding  $R(t)$  values.*

1. A tank contains 50 liters of oil at time  $t = 4$  hours. Oil is being pumped into the tank at a rate  $R(t)$ , where  $R(t)$  is measured in liters per hour, and  $t$  is measured in hours. Selected values of  $R(t)$  are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time  $t = 15$  hours?

- (A) 64.9    (B) 68.2    (C) 114.9    (D) 116.6    (E) 118.2

*oil in the tank @  $t=15$  hrs  $\approx 5.6(3) + 5.9(5) + 6.2(3) + 50$   
 $\approx 114.9$  liters*

*oil in tank @  $t=4$*

*oil added in from  $t=4$  to  $t=15$*

2. A function  $f$  is continuous on  $[1,5]$  and some of the values of  $f$  are shown below:

|        |   |     |   |
|--------|---|-----|---|
| $x$    | 1 | 4   | 5 |
| $f(x)$ | 7 | $b$ | 2 |

*Handwritten notes above table:  $\Delta x=3$  (between 1 and 4),  $\Delta x=1$  (between 4 and 5). Arrows point from these intervals to the corresponding  $f(x)$  values.*

If the right Riemann sum is 17, then the value of  $b$  is:

- (A) -4    (B) 0    (C) 3    (D) 4.25    (E) 5

*right sum =  $1(2) + 3(b)$*

*$17 = 2 + 3b$*

*$15 = 3b$*

*$5 = b$*

**(E) 5**

3. The rate of temperature of water in a tub at time  $t$  is modeled by a strictly decreasing, twice-differentiable function  $W'(t)$ , where  $W'(t)$  is measured in degrees Fahrenheit per minute and  $t$  is measure in minutes. For  $0 \leq t \leq 20$ , use a left Riemann sum with the four subintervals indicated by the data in the table to approximate the temperature of the water over these 20 minutes. Does this approximation overestimate or underestimate the actual temperature of the water?

|   |      |     |     |     |     |
|---|------|-----|-----|-----|-----|
| $t$ (minutes)                             | 0    | 4   | 9   | 15  | 20  |
| $W'(t)$ ( $^{\circ}\text{F}/\text{min}$ ) | 12.1 | 9.7 | 6.1 | 3.1 | 2.3 |

*Handwritten notes above table:  $\Delta t=4$  (between 0 and 4),  $\Delta t=5$  (between 4 and 9),  $\Delta t=6$  (between 9 and 15),  $\Delta t=5$  (between 15 and 20). Arrows point from these intervals to the corresponding  $W'(t)$  values.*

*Temp of water over 20 min  $\approx 4(12.1) + 5(9.7) + 6(6.1) + 5(3.1)$   
 $\approx 149^{\circ}\text{F}$*

*This approximation overestimates the actual temp of water b/c  $W'(t)$  is decreasing*

*units  $(W'(t) \times t)$   
 $(^{\circ}\text{F}/\text{min}) \times \text{min}$*