Date________________

Solving Trigonometric Equations

Classify each of the following as being either: A) an expression to simplify, B) an equation that is never true, C) an equation that is sometimes true, or D) an equation that is always true (an identity).

$\boxed{C}$	$2\sin x = \cos x \sec x$ $2\sin x = \cos x \cdot \frac{1}{\cos x}$ $2\sin x = 1$ $\sin x = 1/2 \quad x = \pi/6, 5\pi/6$	$\boxed{D}$	$\cos x \tan x = \sin x$ $\cos x \cdot \frac{\sin x}{\cos x} = \sin x$ $\sin x = \sin x$
$\boxed{A}$	$\sin x \cot x$ $= \sin x \cdot \frac{\cos x}{\sin x}$ $= \cos x$	$\boxed{B}$	$\sin x - \cos x \sec x = \tan x \cot x$ $\sin x - \cos x \cdot \frac{1}{\cos x} = \tan x \cdot \frac{1}{\tan x}$ $\sin x - 1 = 1$ $\sin x = 2$

By simplifying expressions on both sides of the equal sign, solve each equation on the given interval, or state that the equation is an identity.

A. Solve algebraically. Use inverse operations to solve each equation on the interval $[0, 2\pi)$.

1) $2\sin x = 1$

$\sin x = \frac{1}{2}$

$\boxed{x = \pi/6, 5\pi/6}$

2) $\tan x - 1 = 0$

$\tan x = 1$

$\boxed{x = \pi/4, 5\pi/4}$

$\frac{\sin x}{\cos x} = 1$

$\sin x = \cos x$

3) $4\cos^{2}(x - 1) = 3$

$\sqrt{\cos^2(x-1)} = \sqrt{\frac{3}{4}}$

$\cos(x-1) = \frac{\sqrt{3}}{2}$

$x - 1 = \pi/6 \qquad x - 1 = \frac{11\pi}{6}$

$\boxed{x = \pi/6 + 1} \qquad \boxed{x = \frac{11\pi}{6} + 1}$

4) $2\cos^2 x - 1 = 0$

$2\cos^2 x = 1$

$\cos^2 x = \frac{1}{2}$

$\cos x = \sqrt{1/2}$

$\cos x = \frac{\sqrt{2}}{2}$

$\boxed{x = \pi/4, 7\pi/4}$

B. Simplify and solve. Simplify each side of the equation, then solve on the interval $[0, 2\pi)$.

Solving Tip:

- Sometimes, when multiplying or dividing by an expression on both sides of an equation, extraneous solutions can be introduced. Be sure to check your final solutions.

5) $\cot x = \sqrt{3}$

$\frac{\cos x}{\sin x} = \sqrt{3}$

$\frac{\cos x}{\sin x} = \frac{\sqrt{3}/2}{1/2}$ or $\frac{-\sqrt{3}/2}{-1/2}$

$\boxed{x = \pi/6, 7\pi/6}$

6) $\sin x \cot x = 0$

$\sin x \cdot \frac{\cos x}{\sin x} = 0$ or $\sin x = 0$

$\cos x = 0$

$\boxed{x = \pi/2, 3\pi/2}$ $\boxed{x = 0, \pi}$

7) $\cos x \sec x = 1$

$\cos x \cdot \frac{1}{\cos x} = 1$

$1 = 1$ always true

$\boxed{\text{all reals \#s on } [0, 2\pi)}$

8) $2\cos x \tan x - \sin x \csc x = 0$

$2\cos x \frac{\sin x}{\cos x} - \sin x \cdot \frac{1}{\sin x} = 0$

$2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$\boxed{x = \pi/6, 5\pi/6}$

9) $\cos x \tan x \csc x = 1$

$\cos x \cdot \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} = 1$

$1 = 1$

$\boxed{\text{all real \#s on } [0, 2\pi)}$

10) $\cos x \tan x - \sin x \csc x = 0$

$\cos x \cdot \frac{\sin x}{\cos x} - \sin x \cdot \frac{1}{\sin x} = 0$

$\sin x - 1 = 0$

$\sin x = 1$ $\boxed{x = \pi/2}$

C. Factor and solve.

Set one side of the equation equal to zero; then factor to solve.

Solving Tips:

- Always look for a common factor first.
- If no common factor is present, you may have to use an identity to rewrite the expression in terms of one function (with the same argument). (For example, the identity $\sin^2 x = 1 - \cos^2 x$ can be used to replace sine expressions with cosines.)
- If an expression to be factored has both linear and squared terms (such as $\sin^2 x$ and $\sin x$, or $\cos^2 x$ and $\cos x$), it may make the factoring easier if you replace the trigonometric function with a single letter. (See below.) Just remember to substitute back!

Sample: Solve $2\sin^2 x + 5\sin x - 3 = 0$ $\rightarrow$

Let $y = \sin x$. The equation becomes:

$2y^2 + 5y - 3 = 0$

$(2y - 1)(y + 3) = 0$

$y = \frac{1}{2}$, or $y = -3$

So, $\sin x = \frac{1}{2}$, or $\sin x = -3$ $\leftarrow$

Only acceptable factor, $\sin x = \frac{1}{2}$

$x = \{\pi/6, 5\pi/6\}$

Solve on the interval $[0, 2\pi)$.

11) $2\sin x \cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ $\quad$ $2\sin x - 1 = 0$

$\boxed{x = \pi/2, 3\pi/2}$ $\quad$ $2\sin x = 1$

$\sin x = 1/2$

$\boxed{x = \pi/6, 5\pi/6}$

12) $\tan x \sec x = \tan x$

$\tan x \sec x - \tan x = 0$

$\tan x(\sec x - 1) = 0$

$\tan x = 0$ $\quad$ $\sec x - 1 = 0$

$\frac{\sin x}{\cos x} = 0$ $\quad$ $\sec x = 1$

$\boxed{x = 0, \pi}$ $\quad$ $\frac{1}{\cos x} = 1 \rightarrow \cos x = 1$

$x = 0$

13) $4\cos^2 x - 1 = 0$

$(2\cos x - 1)(2\cos x + 1) = 0$

$2\cos x - 1 = 0$ $\quad$ $2\cos x + 1 = 0$

$\cos x = 1/2$ $\quad$ $\cos x = -1/2$

$\boxed{x = \pi/3, 5\pi/3}$ $\quad$ $\boxed{x = 2\pi/3, 4\pi/3}$

14) $2\sin^2 x + \sin x = 0$

$\sin x(2\sin x + 1) = 0$

$\sin x = 0$ $\quad$ $2\sin x + 1 = 0$

$\sin x = -1/2$

$\boxed{x = 0, \pi}$ $\quad$ $\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

15) $2\cos^2 x + \cos x = 1$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$ $\quad$ $\cos x + 1 = 0$

$\cos x = 1/2$ $\quad$ $\cos x = -1$

$\boxed{x = \pi/3, 5\pi/3}$ $\quad$ $\boxed{x = \pi}$

16) $\sin^2 x - \sin x - 6 = 0$

$u^2 - u - 6 = 0$ $\quad$ $u = \sin x$

$(u - 3)(u + 2) = 0$

$u - 3 = 0$ $\quad$ $u + 2 = 0$

$u = 3$ $\quad$ $u = -2$

$\sin x = 3$ $\quad$ $\sin x = -2$

$\boxed{\text{no solution}}$

17) $\cos^2 x + 3\sin x + 3 = 0$

$1 - \sin^2 x + 3\sin x + 3 = 0$

$-\sin^2 x + 3\sin x + 4 = 0$

$-(\sin^2 x - 3\sin x - 4) = 0$

$-(\sin x - 4)(\sin x + 1) = 0$

$= -1$

$\boxed{x = 3\pi/2}$

18) $\sec^2 x - 2\tan x = 4$

$1 + \tan^2 x - 2\tan x = 4$

$\tan^2 x - 2\tan x - 3 = 0$

$u^2 - 2u - 3 = 0$ $\quad$ $u = \tan x$

$(u - 3)(u + 1) = 0$

$u = 3, \; u = -1$ $\rightarrow$ $\tan x = 3$ $\quad$ $\tan x = -1$

$\boxed{x = \frac{3\pi}{4}, \frac{7\pi}{4}}$