

1. Approximate the definite integral using left Riemann sum, right Riemann sum, and midpoint Riemann sum with 4 equal subintervals.

$$\int_0^2 x\sqrt{3-x} dx$$

$$\Delta x = \frac{2-0}{4} = \frac{1}{2}$$

x	0	1/2	1	1.5	2
f(x)	0	.791	$\sqrt{2}$	1.837	2

$$\text{left} = \frac{1}{2} (0 + .791 + \sqrt{2} + 1.837) = 2.021$$

$$\text{right} = \frac{1}{2} (2 + 1.837 + \sqrt{2} + .791) = 3.021$$

$$\text{midpt} = \frac{1}{2} (.415 + 1.125 + 1.654 + 1.957) = 2.575$$

x	1/4	3/4	1.25	1.75
f(x)	.415	1.125	1.654	1.957

Don't round until final answer ... 😊

2. Approximate  $\int_0^8 f(x) dx$  using a right Riemann sum of four subintervals.

Is this an overapproximation? Explain.

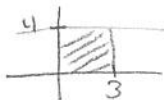
x	0	1	3	7	8
f(x)	0	4	18	70	88

$$\begin{aligned} \int_0^8 f(x) dx &= 1(88) + 4(70) + 2(18) + 1(4) \\ &= 88 + 280 + 36 + 4 \\ &= 408 \end{aligned}$$

overapproximation b/c f(x) inc on (0,8)  
for right Riemann sum

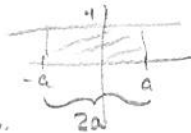
3. Sketch the region whose area is given by the definite integral. Then evaluate the integral.

a)  $\int_0^3 4 dx$



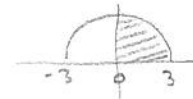
$$3(4) = \boxed{12}$$

b)  $\int_{-a}^a 4 dx$



$$2a(4) = \boxed{8a}$$

c)  $\int_0^3 \sqrt{9-x^2} dx$



$$\frac{\pi(3)^2}{4} = \boxed{\frac{9\pi}{4}}$$

4. If  $\int_2^4 x^3 dx = 60$ ,  $\int_2^4 x dx = 6$ , and  $\int_2^4 dx = 2$ , then evaluate the integrals.

a)  $\int_4^2 x dx$

$$-\int_2^4 x dx = \boxed{-6}$$

b)  $\int_2^4 4x dx$

$$\begin{aligned} &= 4 \int_2^4 x dx \\ &= 4(6) = \boxed{24} \end{aligned}$$

c)  $\int_2^4 (x^3 + 4x) dx$

$$\begin{aligned} &= \int_2^4 x^3 dx + 4 \int_2^4 x dx \\ &= 60 + 4(6) \\ &= \boxed{84} \end{aligned}$$