DATE:_____

Definite Integrals

Riemann Sums – area under curve found by summing up the area of rectangles

Area $\approx \sum$ area of rectangles

$\approx \sum (\text{base})(\text{height})$

$\approx \sum_{k=1}^{n} (\Delta x_k)(f(c_k))$

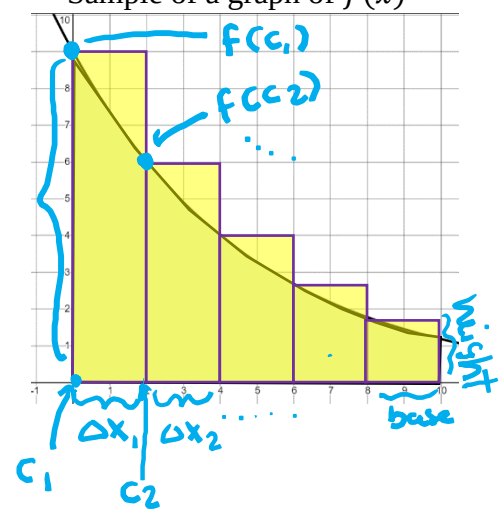
$\approx \sum_{k=1}^{n} f(c_k)\,\Delta x_k$

$n \to$ # of rectangles (subintervals)

$\Delta x_k \to$ width of kth rectangle

$f(c_k) \to$ height of kth rectangle

Sample of a graph of $f(x)$



What if the # of rectangles was infinite?

https://animated-mathematics.net/riemann-sums.html

Area $= \lim_{n\to\infty} \sum_{k=1}^{n} f(c_k)\Delta x_k$

if $n \to \infty$, then $\Delta x \to 0$

$= \lim_{\Delta x \to 0} \sum_{k=1}^{n} f(c_k)\Delta x_k$

Definition of the Definite Integral

If f is defined on $[a, b]$ and $\lim_{\Delta x\to 0} \sum_{k=1}^{n} f(c_k)\Delta x_k$ exists,

then f is integrable on $[a, b]$ and

able to write integral

$$\lim_{\Delta x\to 0} \sum_{k=1}^{n} f(c_k)\Delta x_k = \int_a^b f(x)\,dx$$

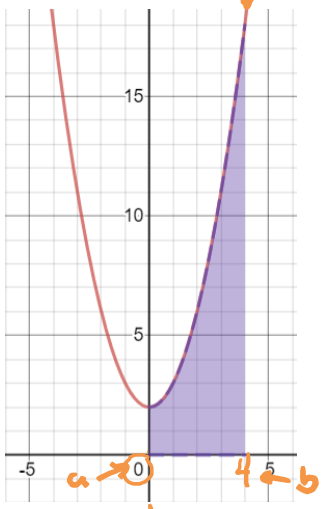
$\int_a^b f(x)\,dx$

"integral from a to b of f(x) dx"

Set up a definite integral that yields the area of the shaded region.

Example 1:

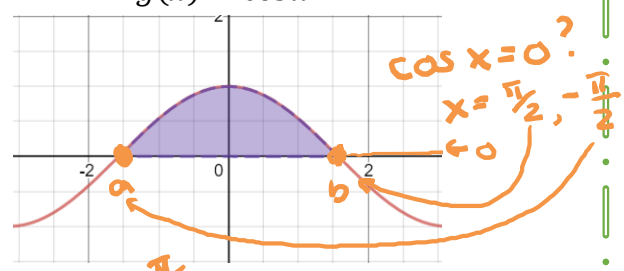
$f(x) = x^2 + 2$



$$\text{Area} = \int_0^4 (x^2+2)\,dx$$

Example 2:

$g(x) = \cos x$



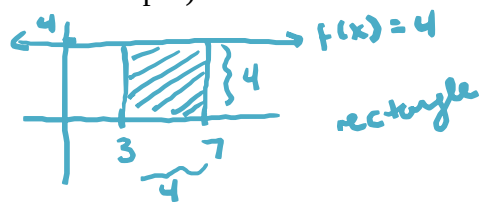
$\cos x = 0$?

$x = \frac{\pi}{2}, -\frac{\pi}{2}$

$$\text{Area} = \int_{-\pi/2}^{\pi/2} \cos x\,dx$$

Evaluate the integral (using knowledge of geometric shapes)

Example 1: $\int_3^7 4\,dx$ $\quad f(x) = 4$



$$\int_3^7 4\,dx = 4(4)$$
$$= \boxed{16}$$

Example 2: $\int_1^3 (-2x+4)\,dx$ $\quad f(x)$

$y = -2x + 4$

slope y-int

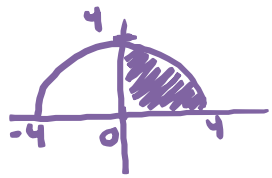
shade btn Function and x-axis

$$\int_1^3 (-2x+4)\,dx = \tfrac{1}{2}(1)(2) + \tfrac{1}{2}(1)(-2)$$
$$= 1 - 1$$
$$= \boxed{0}$$

Evaluate the integral (using knowledge of geometric shapes)

Example 3: $\int_0^4 \sqrt{16 - x^2}\, dx$

b, a, f(x)



$y = \sqrt{16 - x^2}$ only positive

$y^2 = 16 - x^2$

$x^2 + y^2 = 16$ circle w/ radius 4

$\frac{1}{4}$ of a circle

$$\int_0^4 \sqrt{16 - x^2}\, dx = \frac{1}{4}\pi(4)^2$$

$$= \boxed{4\pi}$$

Example 4: $\int_{-1}^{1} (1 - |x|)\, dx$

b, a, f(x)

triangle

$y = 1 - |x|$

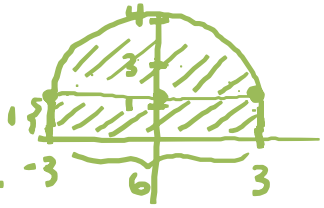
shift up one unit

$$\int_{-1}^{1} (1 - |x|)\, dx = \frac{1}{2}(2)(1)$$

$$= \boxed{1}$$

Example 5: $\int_{-3}^{3} (1 + \sqrt{9 - x^2})\, dx$

f(x)



rectangle and semicircle

$y = \sqrt{9 - x^2}$

$y^2 = 9 - x^2$

$x^2 + y^2 = 9$

semicircle w/ radius 3

$f(x) = 1 + \sqrt{9 - x^2}$

shift up one unit

semicircle

$$\int_{-3}^{3} (1 + \sqrt{9 - x^2})\, dx = 6(1) + \frac{1}{2}\pi(3)^2$$

$$= \boxed{6 + \frac{9\pi}{2}}$$