5.2 Proving Trigonometric Identities

Target 5B: Prove trigonometric identities

Guidelines for Proving Identities

1. It is usually best to work on the more complicated side first.
2. Look for trigonometric substitutions involving the basic identities that may help simplify things.
3. Look for algebraic operations, such as adding fractions, the distributive property, or factoring that may simplify the side you are working with or that will at least lead to an expression that will be easier to simplify.
4. If you cannot think of anything else to do, change everything to sines and cosines and see if that helps.
5. Always keep an eye on the side you are not working with to be sure you are working toward it. There is a certain sense of direction that accompanies a successful proof.
6. Practice, practice, practice!

Prove the identity.

1. $(\sin x)(\cot x + \cos x \tan x) = \cos x + \sin^2 x$

$$\sin x(\cot x + \cos x \tan x) = \sin x\left(\frac{\cos x}{\sin x} + \cancel{\cos x}\cdot\frac{\sin x}{\cancel{\cos x}}\right)$$

$$= \sin x\left(\frac{\cos x}{\sin x} + \sin x\right)$$ "Distribute $\sin x$"

$$= \cancel{\sin x}\cdot\frac{\cos x}{\cancel{\sin x}} + \sin^2 x$$

$$= \cos x + \sin^2 x$$

$\therefore$, $\sin x(\cot x + \cos x \tan x) = \cos x + \sin^2 x$.

2. $\tan\alpha + \sec\alpha = \frac{\cos\alpha}{1-\sin\alpha}$

Proof: $\tan\alpha + \sec\alpha = \frac{\sin\alpha}{\cos\alpha} + \frac{1}{\cos\alpha}$

$$= \frac{\sin\alpha + 1}{\cos\alpha}$$ "multiply by 1" i.e., $\frac{\cos\alpha}{\cos\alpha}$

$$= \frac{\cos\alpha}{\cos\alpha}\cdot\frac{(\sin\alpha+1)}{\cos\alpha}$$

$$= \frac{\cos\alpha(\sin\alpha+1)}{\cos^2\alpha}$$

$$= \frac{\cos\alpha(\sin\alpha+1)}{1-\sin^2\alpha} = \frac{\cos\alpha(\cancel{\sin\alpha+1})}{(1-\sin\alpha)(\cancel{1+\sin\alpha})} = \frac{\cos\alpha}{1-\sin\alpha}$$

$\therefore$, $\tan\alpha + \sec\alpha = \frac{\cos\alpha}{1-\sin\alpha}$.

3. $\frac{\sec^2\beta - 1}{\sin\beta} = \frac{\sin\beta}{1-\sin^2\beta}$

Proof:

$$\frac{\sec^2\beta - 1}{\sin\beta} = \frac{\tan^2\beta}{\sin\beta}$$

$$= \frac{\frac{\sin^2\beta}{\cos^2\beta}}{\sin\beta}$$

$\frac{\sin\beta\sin\beta}{\sin\beta}$

$$= \frac{\sin^2\beta}{\cos^2\beta} \cdot \frac{1}{\cancel{\sin\beta}}$$

$$= \frac{\sin\beta}{\cos^2\beta}$$

$$= \frac{\sin\beta}{1-\sin^2\beta}$$

$\therefore$, $\frac{\sec^2\beta - 1}{\sin\beta} = \frac{\sin\beta}{1-\sin^2\beta}$.

4. $\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = 2\csc x$

Proof:

"Distribute"

$$\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = \frac{\sin x}{(1+\cos x)} \cdot \frac{\sin x}{\sin x} + \frac{(1+\cos x)}{\sin x} \cdot \frac{(1+\cos x)}{(1+\cos x)}$$

$$= \frac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{\sin x(1+\cos x)}$$

$$= \frac{\sin^2 x + \cos^2 x + 1 + 2\cos x}{\sin x(1+\cos x)}$$

"Rearrange numerator"

$$= \frac{1 + 1 + 2\cos x}{\sin x(1+\cos x)}$$

$$= \frac{2 + 2\cos x}{\sin x(1+\cos x)}$$

$$= \frac{2\cancel{(1+\cos x)}}{\sin x\cancel{(1+\cos x)}}$$

$$= 2 \cdot \frac{1}{\sin x}$$

$$= 2\csc x$$

$\therefore$, $\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = 2\csc x$.

More Practice

Proving Trig Identities

http://www.intmath.com/analytic-trigonometry/1-trigonometric-identities.php
http://www.vitutor.com/geometry/trigonometry/identities_problems.html
https://www.youtube.com/watch?v=QGk8sYck_ZI
https://www.youtube.com/watch?v=ep5vjIY5kqE
https://www.youtube.com/watch?v=IE8q4WRubC4

Homework Assignment

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