

Graph of  $g(x)$

1. Let  $g$  be the function, given by the graph above, defined on the closed interval  $-3 \leq x \leq 4$  which consists of one line segment and a semicircle.

Let  $w(x) = \int_{-3}^x g(t) dt$ . Find  $w(-3)$  and  $w(0)$ .

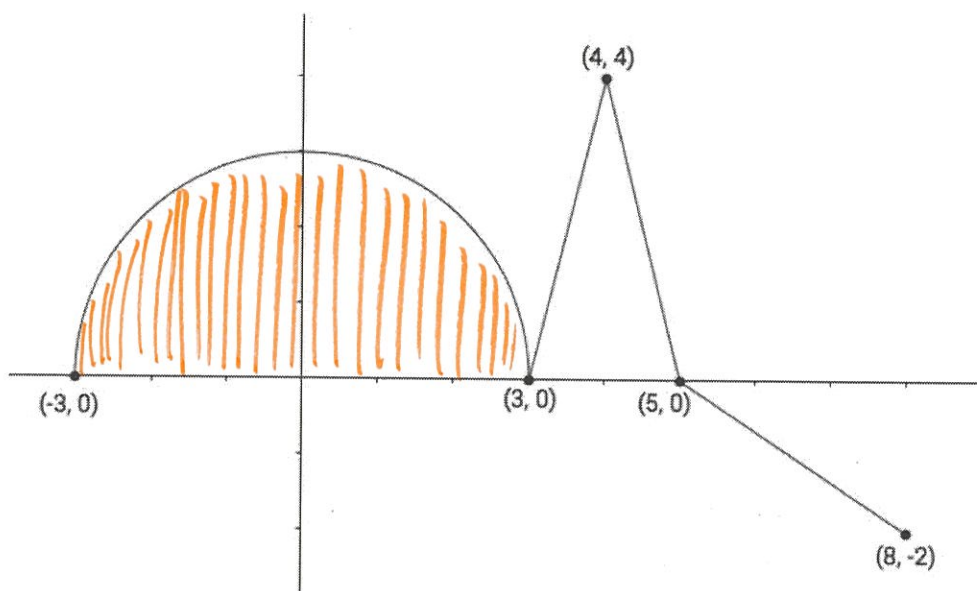
$$w(-3) = \int_{-3}^{-3} g(t) dt$$

$$w(-3) = 0$$

$$w(0) = \int_{-3}^0 g(t) dt = \frac{1}{2}(1)(1) + \frac{1}{2}(2)(-2)$$

$$= \frac{1}{2} - 2$$

$$w(0) = -\frac{3}{2}$$



Graph of  $f(x)$

2. The function  $f$  is defined on the closed interval  $[-3, 8]$  and is given by the graph above which consists of three line segments and a semicircle.

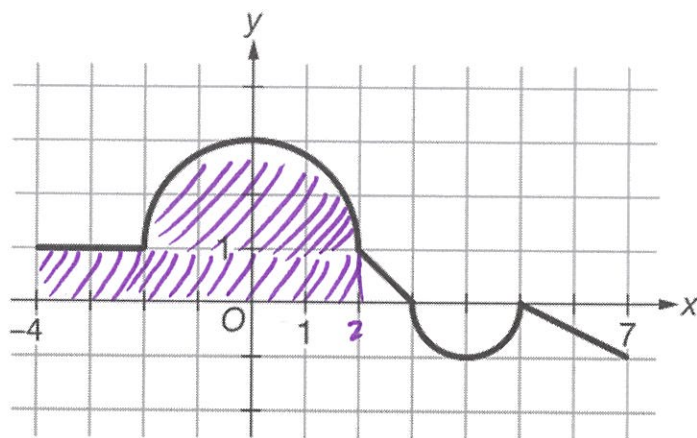
Let  $h$  be the function defined by  $h(x) = x - \int_{x-3}^3 f(t) dt$ . Find  $h(0)$ .

$$h(0) = 0 - \int_{-3}^3 f(t) dt$$

$$= - \int_{-3}^3 f(t) dt$$

$$= -\frac{1}{2}\pi(3)^2$$

$$h(0) = -\frac{9\pi}{2}$$



Graph of  $h(x)$

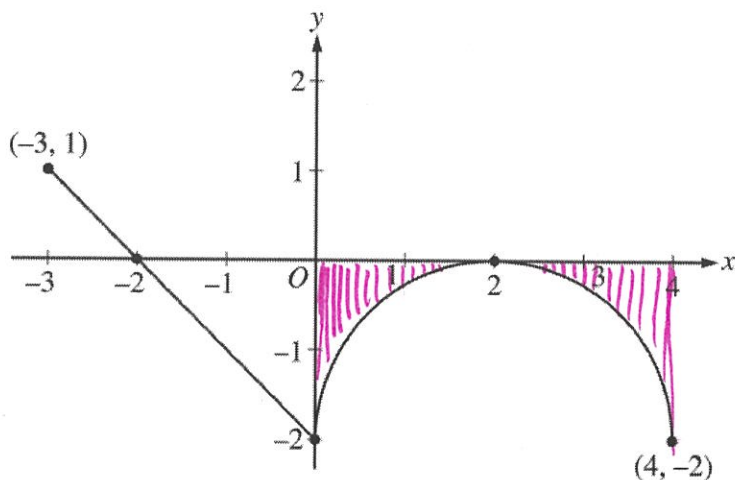
3. The function  $h$  is defined on the closed interval  $[-4, 7]$  and is given by the graph above which consists of three line segments and two semicircles.

Let  $f$  be the function defined by  $f(x) = \int_x^2 h(t) dt$ . Find  $f(-4)$ .

$$f(-4) = \int_{-4}^2 h(t) dt$$

$$= 6(1) + \frac{1}{2}\pi(2)^2$$

$$f(-4) = 6 + 2\pi$$



Graph of  $g(x)$

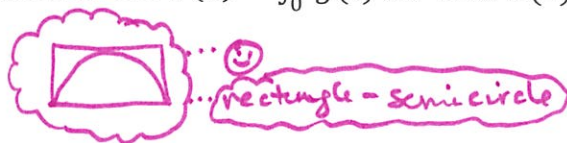
4. Let  $g$  be the function, given by the graph above, defined on the closed interval  $-3 \leq x \leq 4$  which consists of one line segment and a semicircle. Let  $w(x) = \int_0^x g(t) dt$ . Find  $w(4)$ .

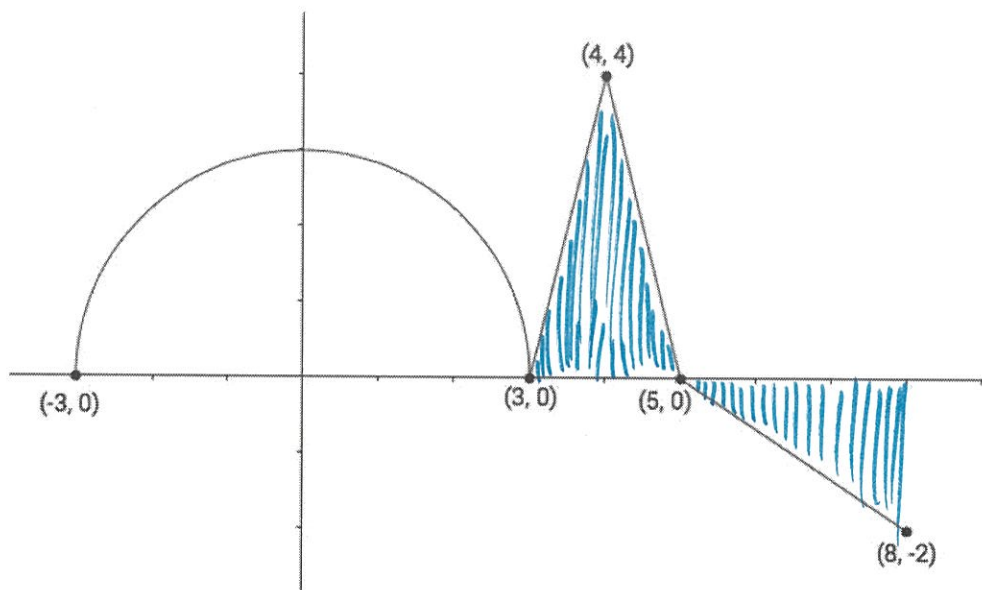
$$w(4) = \int_0^4 g(t) dt$$

$$= -\left(4(2) - \frac{1}{2}\pi(2)^2\right)$$

$$= -(8 - 2\pi)$$

$$w(4) = -8 + 2\pi$$





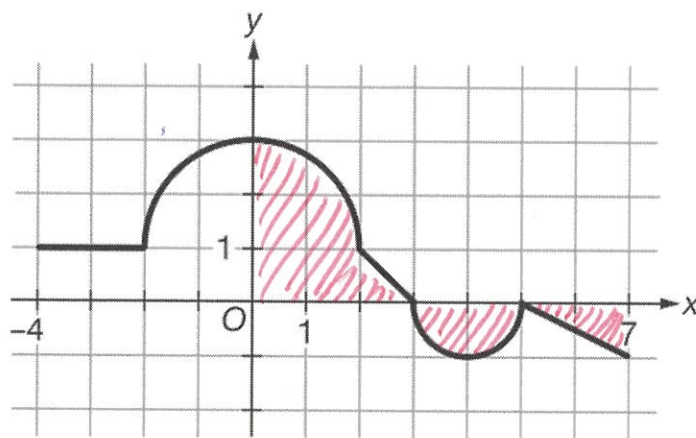
Graph of  $f(x)$

5. The function  $f$  is defined on the closed interval  $[-3, 8]$  and is given by the graph above which consists of three line segments and a semicircle.

Let  $h$  be the function defined by  $h(x) = x - \int_3^{x-4} f(t) dt$ . Find  $h(12)$ .

$$\begin{aligned} h(12) &= 12 - \int_3^8 f(t) dt \\ &= 12 - \left( \frac{1}{2}(2)(4) + \frac{1}{2}(3)(-2) \right) \\ &= 12 - (4 - 3) \\ &= 12 - 1 \end{aligned}$$

$$\boxed{h(12) = 11}$$



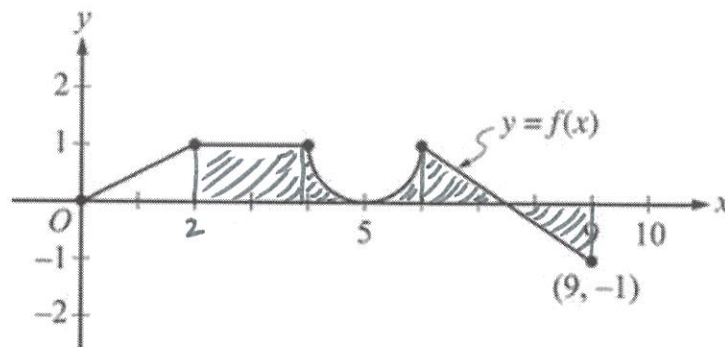
Graph of  $h(x)$

6. The function  $h$  is defined on the closed interval  $[-4, 7]$  and is given by the graph above which consists of three line segments and two semicircles.

Let  $f$  be the function defined by  $f(x) = \int_x^7 h(t) dt$ . Find  $f(0)$ .

$$\begin{aligned} f(0) &= \int_0^7 h(t) dt \\ &= \frac{1}{4}\pi(2)^2 + 2 + \frac{1}{2}(1)(1) - \frac{1}{2}\pi(1)^2 + \frac{1}{2}(2)(-1) \\ &= \pi + 2 + \frac{1}{2} - \frac{1}{2}\pi - 1 \end{aligned}$$

$$\begin{aligned} \rightarrow f(0) &= \pi + 2 + \frac{1}{2} - \frac{1}{2}\pi - 1 \\ f(0) &= \frac{\pi}{2} + \frac{3}{2} \end{aligned}$$

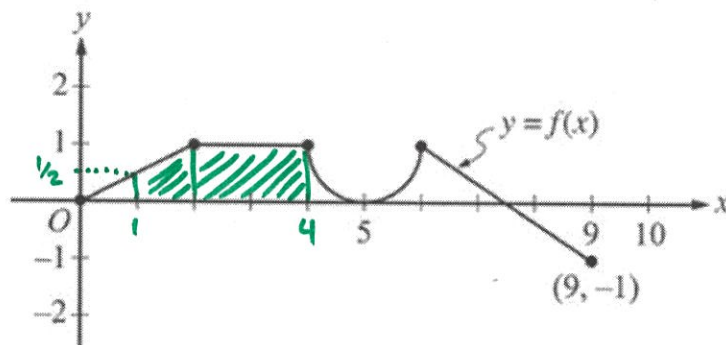


7. The function  $f$  is defined on the closed interval  $[0, 9]$  and is given by the graph above which consists of three line segments and a semicircle centered at point  $(5, 1)$ .

Let  $g$  be the function defined by  $g(x) = \int_2^x f(t) dt$ . Find  $g(9)$ .

$$\begin{aligned}
 g(9) &= \int_2^9 f(t) dt \\
 &= 2(1) + 2(1) - \frac{1}{2}\pi(1)^2 + \frac{1}{2}(1.5)(1) + \frac{1}{2}(1.5)(-1) \\
 &= 2 + 2 - \frac{\pi}{2} \\
 \boxed{g(9) = 4 - \frac{\pi}{2}}
 \end{aligned}$$

rectangle - semicircle ... ☺



8. The function  $f$  is defined on the closed interval  $[0, 9]$  and is given by the graph above which consists of three line segments and a semicircle centered at point  $(5, 1)$ .

Let  $g$  be the function defined by  $g(x) = \int_1^x f(t) dt$ . Find  $g(4)$ .

$$\begin{aligned}
 g(4) &= \int_1^4 f(t) dt \\
 &= \frac{1}{2}(1)\left(\frac{1}{2} + 1\right) + 2(1) \\
 &= \frac{1}{2}\left(\frac{3}{2}\right) + 2 \\
 &= \frac{3}{4} + 2 \\
 \boxed{g(4) = \frac{11}{4}}
 \end{aligned}$$

trapezoid ... ☺ or triangle rectangle