DATE: __________

5.2 Proving Identities

Target 6B: Prove Trigonometric Identities

With your group members, prove the following identities. Your group will prove one to the class.

Prove the identity.

1. $\frac{1}{\tan\theta} + \tan\theta = \sec\theta\csc\theta$

Pf: $\frac{1}{\tan\theta} + \tan\theta = \cot\theta + \tan\theta$

$= \frac{\cos\theta}{\sin\theta}\cdot 1 + \frac{\sin\theta}{\cos\theta}\cdot 1$

$= \frac{\cos\theta}{\sin\theta}\cdot\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\cdot\frac{\sin\theta}{\sin\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta\cdot\sin\theta}$

$= \frac{1}{\cos\theta\cdot\sin\theta}$

$= \frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}$

$= \sec\theta\cdot\csc\theta$

$\therefore$, $\frac{1}{\tan\theta} + \tan\theta = \sec\theta\cdot\csc\theta$

Another approach:

Pf 2: $\frac{1}{\tan\theta} + \tan\theta = \frac{1}{\tan\theta} + \frac{\tan\theta}{1}\cdot\frac{\tan\theta}{\tan\theta}$

$= \frac{1+\tan^2\theta}{\tan\theta} = \frac{\sec^2\theta}{\tan\theta} = \frac{\frac{1}{\cos^2\theta}}{\frac{\sin\theta}{\cos\theta}}$

$= \frac{\cos\theta}{\cos^2\theta\cdot\sin\theta}$

$= \frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}$

$= \sec\theta\cdot\csc\theta$

2. $2\csc^2\alpha = \frac{1}{1-\cos\alpha} + \frac{1}{1+\cos\alpha}$

Go from right to left!

Pf: $\frac{1}{1-\cos\alpha} + \frac{1}{1+\cos\alpha} = \frac{1}{(1-\cos\alpha)}\cdot\frac{(1+\cos\alpha)}{(1+\cos\alpha)} + \frac{1}{(1+\cos\alpha)}\cdot\frac{(1-\cos\alpha)}{(1-\cos\alpha)}$

$= \frac{1+\cancel{\cos\alpha}+1-\cancel{\cos\alpha}}{(1-\cos\alpha)(1+\cos\alpha)}$

$= \frac{2}{1+\cancel{\cos\alpha}-\cancel{\cos\alpha}-\cos^2\alpha}$

$= \frac{2}{1-\cos^2\alpha}$

$= \frac{2}{\sin^2\alpha}$

$= 2\left(\frac{1}{\sin^2\alpha}\right)$

$= 2\csc^2\alpha$

$\therefore$, $\frac{1}{1-\cos\alpha} + \frac{1}{1+\cos\alpha} = 2\csc^2\alpha$.

3. $\frac{\sec x+1}{\tan x} = \frac{\sin x}{1-\cos x}$

Pf: $\frac{\sec x+1}{\tan x} = \frac{\frac{1}{\cos x}+1}{\frac{\sin x}{\cos x}}$

$= \frac{\frac{1}{\cos x}+\frac{\cos x}{\cos x}}{\frac{\sin x}{\cos x}}$

$= \frac{\frac{1+\cos x}{\cos x}}{\frac{\sin x}{\cos x}}$

$= \left(\frac{1+\cos x}{\cancel{\cos x}}\right)\cdot\frac{\cancel{\cos x}}{\sin x}$

$= \frac{(1+\cos x)}{\sin x}\cdot 1$ "Need sinx in numerator"

$= \frac{(1+\cos x)(1-\cos x)}{\sin x\,(1-\cos x)}$

$= \frac{1-\cos^2 x}{\sin x\,(1-\cos x)}$

$= \frac{\sin^2 x}{\sin x\,(1-\cos x)} = \frac{\sin x}{1-\cos x}$

$\therefore$, $\frac{\sec x+1}{\tan x} = \frac{\sin x}{1-\cos x}$.

4. $\frac{\cot y-1}{\cot y+1} = \frac{1-\tan y}{1+\tan y}$

Pf: $\frac{\cot y-1}{\cot y+1} = \frac{\frac{1}{\tan y}-1}{\frac{1}{\tan y}+1}$

$= \frac{\left(\frac{1}{\tan y}-1\right)}{\left(\frac{1}{\tan y}+1\right)}\cdot 1$

$= \frac{\left(\frac{1}{\tan y}-1\right)}{\left(\frac{1}{\tan y}+1\right)}\cdot\frac{\tan y}{\tan y}$ "Distribute" tany

$= \frac{1-\tan y}{1+\tan y}$

$\therefore$, $\frac{\cot y-1}{\cot y+1} = \frac{1-\tan y}{1+\tan y}$.

5. $(\cos x - \sin x)^2 = 1 - 2\sin x \cos x$

Pf: $(\cos x - \sin x)^2 = (\cos x - \sin x)(\cos x - \sin x)$

$= \cos^2 x - \cos x \sin x - \cos x \sin x + \sin^2 x$

$= \underbrace{\cos^2 x + \sin^2 x} - 2\cos x \sin x$

$= 1 - 2\sin x \cos x$

$\therefore$, $(\cos x - \sin x)^2 = 1 - 2\sin x \cos x$.

6. $\tan^2\alpha - \sin^2\alpha = \tan^2\alpha \sin^2\alpha$

Pf: $\tan^2\alpha - \sin^2\alpha = \dfrac{\sin^2\alpha}{\cos^2\alpha} - \sin^2\alpha$

$= \dfrac{1}{\cos^2\alpha}\cdot \sin^2\alpha - \sin^2\alpha$

$= \sec^2\alpha \cdot \sin^2\alpha - \sin^2\alpha$

$= \sin^2\alpha(\sec^2\alpha - 1)$

$= \sin^2\alpha \cdot \tan^2\alpha$

$\therefore$, $\tan^2\alpha - \sin^2\alpha = \sin^2\alpha \cdot \tan^2\alpha$.

* The other way around is easy too

7. $\dfrac{1-\cos\beta}{\sin\beta} = \dfrac{\sin\beta}{1+\cos\beta}$

Pf: $\dfrac{1-\cos\beta}{\sin\beta} = \dfrac{1-\cos\beta}{\sin\beta}\cdot 1$

$= \dfrac{(1-\cos\beta)}{\sin\beta}\cdot\dfrac{(1+\cos\beta)}{(1+\cos\beta)}$

$= \dfrac{1-\cos^2\beta}{\sin\beta(1+\cos\beta)}$

$= \dfrac{\sin^2\beta}{\sin\beta(1+\cos\beta)}$

$= \dfrac{\sin\beta}{1+\cos\beta}$

$\therefore$, $\dfrac{1-\cos\beta}{\sin\beta} = \dfrac{\sin\beta}{1+\cos\beta}$.

8. $2\csc\theta = \dfrac{1+\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta}$

Pf: $\dfrac{1+\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{(1+\cos\theta)}{\sin\theta}\cdot\dfrac{(1+\cos\theta)}{(1+\cos\theta)} + \dfrac{\sin\theta}{(1+\cos\theta)}\cdot\dfrac{\sin\theta}{\sin\theta}$

$= \dfrac{1 + 2\cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta(1+\cos\theta)}$

$= \dfrac{1 + 2\cos\theta + 1}{\sin\theta(1+\cos\theta)}$

$= \dfrac{2 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \dfrac{2\cancel{(1+\cos\theta)}}{\sin\theta\cancel{(1+\cos\theta)}}$

$= 2\left(\dfrac{1}{\sin\theta}\right)$

$= 2\csc\theta$

$\therefore$, $\dfrac{1+\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta} = 2\csc\theta$.