

Group members _____

Pass the Trigonometric Proof

Write the Pythagorean Identities

(write one and pass it on- next person checks previous person's work)

1. $\sin^2 x + \cos^2 x = 1$

2. $1 + \tan^2 x = \sec^2 x$

3. $\cot^2 x + 1 = \csc^2 x$

Prove the Identities

(write one step and pass it on- next person checks previous person's work)

1. $\frac{\tan^2 x + 1}{1 + \cot^2 x} = \tan^2 x$

$$\begin{aligned} &= \frac{\sec^2 x}{\csc^2 x} \\ &= \frac{1}{\cos^2 x} \cdot \frac{1}{\frac{1}{\sin^2 x}} \\ &= \frac{\sin^2 x}{\cos^2 x} \\ &= \tan^2 x \end{aligned}$$

2. $\frac{\sin^2 \alpha \cot^2 \alpha}{1 - \sin^2 \alpha} = 1$

$$\begin{aligned} &= \frac{\sin^2 \alpha \cdot \frac{\cos^2 \alpha}{\sin^2 \alpha}}{1 - \sin^2 \alpha} \\ &= \frac{\cos^2 \alpha}{1 - \sin^2 \alpha} \\ &= \frac{\cos^2 \alpha}{\cos^2 \alpha} \\ &= 1 \end{aligned}$$

3. $\frac{\cos^2 x \sin^2 x}{1 - \cot^2 x} = \frac{\cos^2 x}{1 - \cot^2 x}$

$$\begin{aligned} &= \frac{\cos^2 x}{1 - (\csc^2 x - 1)} \\ &= \frac{\cos^2 x}{1 - \csc^2 x + 1} \\ &= \frac{\cos^2 x}{\csc^2 x} \\ &= \cos^2 x \cdot \frac{1}{\sin^2 x} \\ &= \cos^2 x (\sin^2 x) \end{aligned}$$

4. $\sin \theta + \cos \theta = \frac{\tan \theta + 1}{\sec \theta}$

$$\begin{aligned} &= \frac{\frac{\sin \theta}{\cos \theta} + 1}{\frac{1}{\cos \theta}} \\ &= \frac{\sin \theta + \cos \theta}{\frac{\cos \theta}{1}} \\ &= \frac{\sin \theta + \cos \theta}{\cos \theta} \cdot \frac{\cos \theta}{1} \\ &= \sin \theta + \cos \theta \end{aligned}$$

$$5. \frac{\tan x - \tan x \sin^2 x}{2 \sin x \cos x} = \frac{1}{2}$$

$$= \frac{\tan x (1 - \sin^2 x)}{2 \sin x \cos x}$$

$$= \frac{\tan x (\cos^2 x)}{2 \sin x \cos x}$$

$$= \frac{\tan x \cdot \cos x}{2 \sin x}$$

$$= \frac{\frac{\sin x}{\cos x} \cdot \cos x}{2 \sin x}$$

$$= \frac{\sin x}{2 \sin x}$$

$$= \frac{1}{2}$$

$$6. \frac{1 + \cot x}{\csc x} = \sin x + \cos x$$

$$= \frac{1 + \frac{\cos x}{\sin x}}{\frac{1}{\sin x}}$$

$$= \frac{\sin x + \cos x}{\frac{1}{\sin x}}$$

$$= \frac{\sin x + \cos x}{1} \cdot \frac{\sin x}{\sin x}$$

$$= \frac{\sin x + \cos x}{1} \cdot \frac{\sin x}{1}$$

$$= \sin x + \cos x$$

CHALLENGE: As a group, write your own identity for another group to prove.