

Mean Value Theorem for Integrals

Recall MVT...

MEAN VALUE THEOREM

↳ "average rate of change" → "slope" → "derivative"

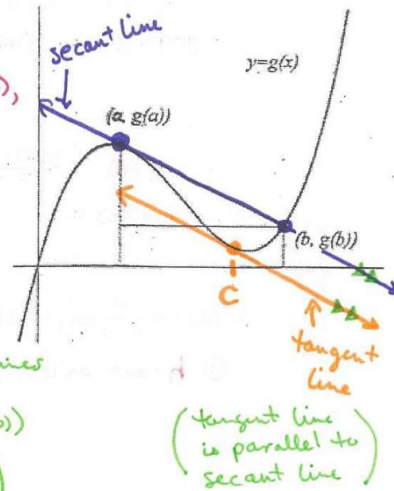
If f is cont on $[a, b]$ and diff'able on (a, b) ,
then \exists a $\#$, c , in (a, b) s.t.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

This means:

- ① f cont on $[a, b]$ → no jumps, holes, or V.A.
- ② f diff'able on (a, b) → no corner, no discont, no vertical tangent lines
- ③ $f'(c) =$ slope of line for $(a, f(a)) + (b, f(b))$

$$\left(\begin{array}{l} \text{slope of} \\ \text{tangent} \\ \text{line} \\ \text{@ } x=c \end{array} \right) = \left(\begin{array}{l} \text{slope of secant line} \\ \text{through } (a, f(a)) + (b, f(b)) \end{array} \right)$$

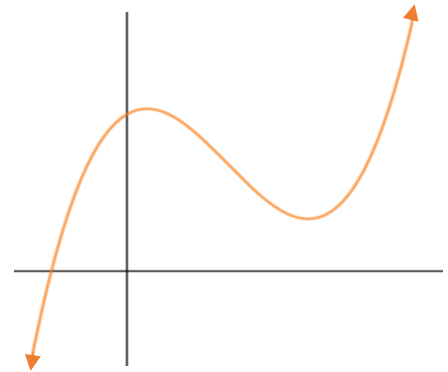


MVT for Integrals

If f is integrable on $[a, b]$,

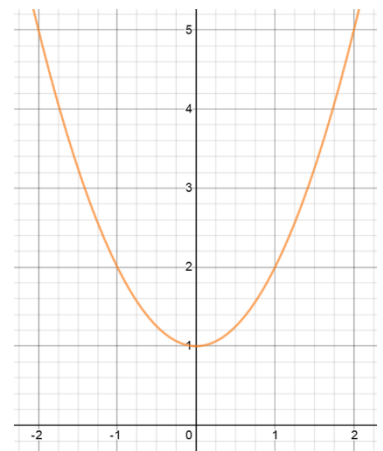
then \exists some c on $[a, b]$ such that

$$\int_a^b f(x) =$$



• Example:

$f(x) = 1 + x^2$ on $[-1, 2]$. Find the value of c that satisfies the Mean Value Theorem for Integrals.



Average Value of a Function

Recall that: value means y -value



If f is integrable on $[a, b]$,

then

Average
Value of a =
Function

• *Example:*

Find the average value of $f(x) = \cos x$ on $\left[0, \frac{\pi}{2}\right]$.

