

Mean Value Theorem for Integrals

Recall MVT...

MEAN VALUE THEOREM

"average rate of change" → "slope" → "derivative"

If f is cont on $[a, b]$ and diff'ble on (a, b) ,
then \exists a #, c , in (a, b) s.t.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

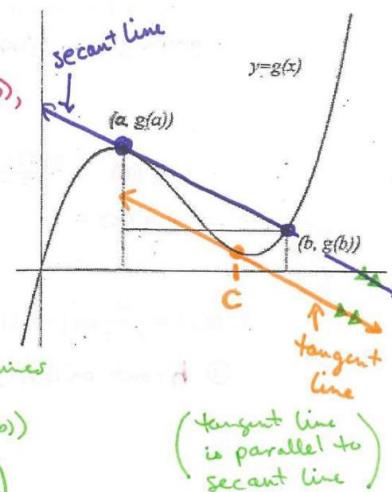
This means:

① f cont on $[a, b] \rightarrow$ no jumps, holes, or V.A.

② f diff'ble on $(a, b) \rightarrow$ no corner, no discord, no vertical tangent lines

③ $f'(c) = \text{slope of line for } (a, f(a)) + (b, f(b))$

$$\left(\begin{array}{l} \text{slope of tangent line} \\ @x=c \end{array} \right) = \left(\begin{array}{l} \text{slope of secant line} \\ \text{through } (a, f(a)) + (b, f(b)) \end{array} \right)$$



MVT for Integrals

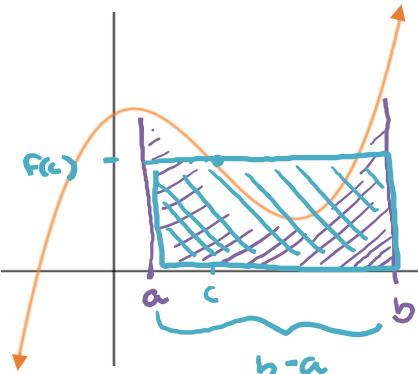
If f is integrable on $[a, b]$,

then \exists some c on $[a, b]$ such that

$$\int_a^b f(x) dx = (b-a) f(c)$$

base height
of some rectangle
 $@ x=c$

area under
 $f(x)$
from a to b
 $\text{area under curve} = \text{area of some rectangle}$



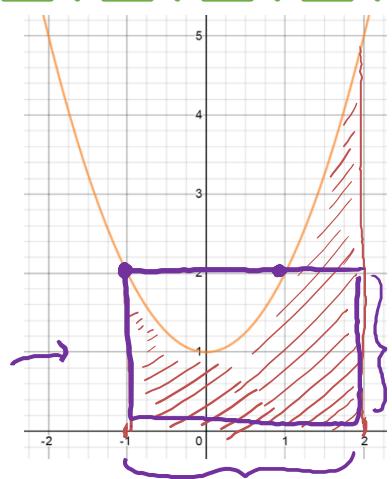
Example:

$f(x) = 1 + x^2$ on $[-1, 2]$. Find the value of c that satisfies the Mean Value Theorem for Integrals.

$$\begin{aligned} \int_{-1}^2 (1+x^2) dx &= (2-(-1)) f(c) \\ (x + \frac{1}{3}x^3) \Big|_{-1}^2 &= 3(1+c^2) \\ 2 + \frac{1}{3}(2)^3 - (-1 + \frac{1}{3}(-1)^3) &= 3 + 3c^2 \\ 2 + \frac{8}{3} + \frac{1}{3} + \frac{1}{3} &= 3 + 3c^2 \\ 3 + 3 &= 3 + 3c^2 \\ 6 &= 3 + 3c^2 \\ 3 &= 3c^2 \\ 1 &= c^2 \\ \pm 1 &= c \end{aligned}$$

area of rectangle
= area under curve

$$3(2) = 6$$



Average Value of a Function

Recall that: value means y -value



If f is integrable on $[a, b]$,

then

$$\text{Average Value of a Function} = \frac{1}{b-a} \int_a^b f(x) dx$$

MVT for Integrals

$$\int_a^b f(x) dx = \frac{b-a}{b-a} \cdot \underline{\int_a^b f(x) dx} = f(c)$$

any value of function

Example:

Find the average value of $f(x) = \cos x$ on $\left[0, \frac{\pi}{2}\right]$.

$$\begin{aligned} \text{any value of } f(x) &= \frac{1}{\frac{\pi}{2}-0} \int_0^{\frac{\pi}{2}} \cos x dx \\ &= \frac{1}{\frac{\pi}{2}} (\sin x) \Big|_0^{\frac{\pi}{2}} \\ &= \frac{2}{\pi} (\sin \frac{\pi}{2} - \sin 0) \\ &= \frac{2}{\pi} (1 - 0) \\ &= \frac{2}{\pi} \end{aligned}$$

