DATE: __________

5.3 Sum and Difference Identities

Target 6B: Prove Trigonometric Identities

Find the exact value of the expression.

1. $\tan(195°) = \tan(150° + 45°)$

$$= \frac{\sin 150° \cos 45° + \cos 150° \sin 45°}{\cos 150° \cos 45° - \sin 150° \sin 45°}$$

$$= \frac{\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{-\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}$$

$$= \frac{\frac{\sqrt{2}-\sqrt{6}}{4}}{\frac{-\sqrt{6}-\sqrt{2}}{4}}$$

$$= \frac{\sqrt{2}-\sqrt{6}}{4} \cdot \frac{4}{-\sqrt{6}-\sqrt{2}}$$

$$= \boxed{\frac{\sqrt{2}-\sqrt{6}}{-\sqrt{6}-\sqrt{2}}} \text{ or } \boxed{-\frac{\sqrt{2}-\sqrt{6}}{\sqrt{6}+\sqrt{2}}}$$

2. $\sin\left(\frac{23\pi}{12}\right) = \sin\left(\frac{15\pi}{12} + \frac{8\pi}{12}\right)$

$$= \sin\left(\frac{5\pi}{4} + \frac{2\pi}{3}\right)$$

$$= \sin\frac{5\pi}{4}\cos\frac{2\pi}{3} + \cos\frac{5\pi}{4}\sin\frac{2\pi}{3}$$

$$= -\frac{\sqrt{2}}{2}\left(-\frac{1}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} + \frac{-\sqrt{6}}{4}$$

$$= \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

3. $\cos\left(-\frac{5\pi}{12}\right) = \cos\left(\frac{3\pi}{12} - \frac{8\pi}{12}\right)$

$$= \cos\left(\frac{\pi}{4} - \frac{2\pi}{3}\right)$$

$$= \cos\frac{\pi}{4}\cos\frac{2\pi}{3} + \sin\frac{\pi}{4}\sin\frac{2\pi}{3}$$

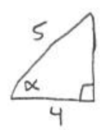
$$= \frac{\sqrt{2}}{2}\left(-\frac{1}{2}\right) + \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{-\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

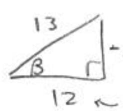
$$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

Evaluate the expression given that $\cos\alpha = \frac{4}{5}$**, where** $\overbrace{0 < \alpha < \frac{\pi}{2}}^{\text{quad I}}$ **and** $\sin\beta = -\frac{5}{13}$**, where** $\overbrace{\frac{3\pi}{2}}^{\text{quad IV}} < \alpha < 2\pi$**.**

3. $\sin(\alpha + \beta)$



$4^2 + a^2 = 5^2$
$a^2 = 9$
$a = 3$



$(-5)^2 + a^2 = 13^2$
$a^2 = 144$
$a = 12$

$$\sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\left(\frac{3}{5}\right)\left(\frac{12}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{-5}{13}\right)$$

$$\frac{36}{65} - \frac{20}{65}$$

$$\boxed{\frac{16}{65}}$$

4. $\tan(\alpha - \beta)$

$$= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

use the same α + β Δs

$$= \frac{\frac{3}{4} - \frac{-5}{12}}{1 + \left(\frac{3}{4}\right)\left(\frac{-5}{12}\right)}$$

reduce

common denominator

$$= \frac{\frac{9}{12} + \frac{5}{12}}{1 + \frac{-5}{16}}$$

$$= \frac{\frac{14}{12}}{\frac{16}{16} - \frac{5}{16}} = \frac{\frac{7}{6}}{\frac{11}{16}} = \frac{7}{6} \cdot \frac{16}{11}$$

$$= \boxed{\frac{56}{33}}$$

Simplify the expression.

5. $\tan(x + \pi)$

$$= \frac{\tan x + \tan\pi}{1 - \tan x \tan\pi}$$

$$= \frac{\tan x + 0}{1 - \tan x(0)}$$

$$= \frac{\tan x}{1}$$

$$= \boxed{\tan x}$$

6. $\cos\left(x - \frac{3\pi}{2}\right)$

$$\cos x \cos\frac{3\pi}{2} + \sin x \sin\frac{3\pi}{2}$$

$$\cos x(0) + \sin x(-1)$$

$$\boxed{-\sin x}$$