DATE: _____

2nd Fundamental Theorem of Calculus

Make a Connection….

1. Given $g(x) = \int_3^x (t^2 - 2)\,dt$, find $g'(x)$.

$= \left(\frac{1}{3}t^3 - 2t\right)\Big|_3^x$

$= \frac{1}{3}x^3 - 2x - \left(\frac{1}{3}(3)^3 - 2(3)\right)$

$= \frac{1}{3}x^3 - 2x - 9 + 6$

$g(x) = \frac{1}{3}x^3 - 2x - 3$

connection?

$g'(x) = \frac{1}{3}(3x^2) - 2$

$g'(x) = x^2 - 2$

2. In problem #1, can you see a connection between $g(x)$ and $g'(x)$? If yes, find $g'(x)$ in one step for each problem below.

a) $g(x) = \int_4^x (3t^2 - 2)\,dt$ **b)** $g(x) = \int_5^x (3t^2 - 2t)\,dt$

$g'(x) = 3x^2 - 2$ $g'(x) = 3x^2 - 2x$

3. For $g(x) = \int_3^{x^2} (t^2 - 2)\,dt$, $g'(x) = 2x^5 - 4x$. Does your method apply to this problem? If not, how can you fix your method?

derive

$= 2x(x^4 - 2)$

$g'(x) = (x^2)^2 - 2) \cdot 2x$

$= (x^4 - 2)\,2x$

$= 2x^5 - 4x$

2nd FTC

If f is continuous on $[a, b]$ and

$F(x) = \int_a^x f(t)\,dt$ $\forall x$ in $[a, b]$ (variable, # constant)

then $F'(x) = \frac{d}{dx}\int_a^x f(t)\,dt$

$F'(x) = f(x)$

and if $F(x) = \int_a^{g(x)} f(t)\,dt$ $\forall x$ in $[a, b]$ (function, # constant)

then $F'(x) = \frac{d}{dx}\int_a^{g(x)} f(t)\,dt$

$F'(x) = f(g(x)) \cdot g'(x)$

Example 1:

Find $F'(x)$ where $F(x) = \int_x^{\pi} \sqrt{1+\sec t}\, dt$.

constant

$= -\int_{\pi}^{x} \sqrt{1+\sec t}\, dt$

$F'(x) = -\frac{d}{dx}\int_{\pi}^{x} \sqrt{1+\sec t}\, dt$

$F'(x) = -\sqrt{1+\sec x}$

Example 2:

If $h(x) = \int_1^{\tan x} \sqrt{t+\sqrt{t}}\, dt$, find $h'(x)$.

$h'(x) = \sqrt{\tan x + \sqrt{\tan x}} \cdot \sec^2 x$

Example 3:

Given $y = \int_{e^x}^{0} \sin^3 t\, dt$, find $\frac{dy}{dx}$.

$= -\int_{0}^{e^x} \sin^3 t\, dt$

$\frac{dy}{dx} = -\sin^3 e^x \cdot e^x \longrightarrow \frac{dy}{dx} = -e^x \sin^3 e^x$

Example 4:

Find $\frac{d}{dx}\int_4^{3x} \cos t\, dt$.

$\frac{d}{dx}\int_4^{3x} \cos t\, dt = \cos 3x \cdot 3$

$= 3\cos 3x$

Example 5:

If $g(x) = \int_{2x}^{3x} \frac{t^2-1}{t^2+1} dt$, find $g'(x)$.

$= \int_{2x}^{c} \frac{t^2-1}{t^2+1} dt + \int_{c}^{3x} \frac{t^2-1}{t^2+1} dt$

$= -\int_{c}^{2x} \frac{t^2-1}{t^2+1} dt + \int_{c}^{3x} \frac{t^2-1}{t^2+1} dt$

$g'(x) = -\frac{(2x)^2-1}{(2x)^2+1} \cdot 2 + \frac{(3x)^2-1}{(3x)^2+1} \cdot 3$

$= -2 \cdot \frac{4x^2-1}{4x^2+1} + 3 \cdot \frac{9x^2-1}{9x^2+1}$

$g'(x) = \frac{-8x^2+2}{4x^2+1} + \frac{27x^2-3}{9x^2+1}$