5.4 Multiple Angle Identities

Target 6C: Solve equations using trigonometric identities

Review Prior Concepts

Solve the trigonometric equation for x on $[0, 2\pi)$.

1) $\cos^2 x - 1 = 0$

$\cos^2 x = 1$

$\cos x = \pm 1$

$\boxed{x = 0, \pi}$

2) $\tan x - \sec x \cos x = 0$

$\tan x - \frac{1}{\cos x} \cdot \cos x = 0$

$\tan x - 1 = 0$

$\tan x = 1$

$\boxed{x = \pi/4, 5\pi/4}$

Double-Angle Identities

$\sin 2A = \sin(A + A)$

$= \sin A \cos A + \cos A \sin A$

$= \sin A \cos A + \sin A \cos A$

$\boxed{\sin 2A = 2 \sin A \cos A}$

$\cos 2A = \cos(A + A)$

$= \cos A \cos A - \sin A \sin A$

$\boxed{\cos 2A = \cos^2 A - \sin^2 A}$

$= \overbrace{1 - \sin^2 A} - \sin^2 A$

$\boxed{\cos 2A = 1 - 2\sin^2 A}$

$\cos 2A = \cos^2 A - \sin^2 A$

$= \cos^2 A - (\overbrace{1 - \cos^2 A})$

$= \cos^2 A - 1 + \cos^2 A$

$\boxed{\cos 2A = 2\cos^2 A - 1}$

$\tan 2A = \tan(A + A)$

$= \frac{\tan A + \tan A}{1 - \tan A \tan A}$

$\boxed{\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}}$

Examples

Solve for x on $[0, 2\pi)$.

1) $\sin 2x = \sin x$

$\sin 2x - \sin x = 0$

$2 \sin x \cos x - \sin x = 0$

$\sin x (2\cos x - 1) = 0$

$\sin x = 0$

$x = 0, \pi$

$2\cos x - 1 = 0$

$2\cos x = 1$

$\cos x = \frac{1}{2}$

$x = \pi/3, 5\pi/3$

* don't forget to check for extraneous solutions

$\boxed{x = 0, \pi, \pi/3, \frac{5\pi}{3}}$

2) $\cos 2x = \cos x$

$\cos 2x - \cos x = 0$

$2\cos^2 x - 1 - \cos x = 0$

$2\cos^2 x - \cos x - 1 = 0$ let $u = \cos x$

$2u^2 - u - 1 = 0$

$(2u+1)(u-1) = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$\cos x - 1 = 0$

$\cos x = 1$

$x = 0$

$\boxed{x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}}$

Prove the identity.

1) $\cos 6x = 2\cos^2 3x - 1$

$\cos(2(3x)) =$

$\cos(2A) =$

$2\cos^2 A - 1 =$

$2\cos^2(3x) - 1 =$

$3x = A$ … ☺

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2) $2\cot 2x = \cot x - \tan x$

$2 \cdot \frac{1}{\tan 2x} =$

$2 \cdot \frac{1}{\frac{2\tan x}{1-\tan^2 x}} =$

$\cancel{2} \cdot \frac{1-\tan^2 x}{\cancel{2}\tan x} =$

$\frac{1-\tan^2 x}{\tan x} =$

$\frac{1}{\tan x} - \frac{\tan^2 x}{\tan x} =$

$\cot x - \tan x =$

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3) $\sin 3x = \sin x\,(3 - 4\sin^2 x)$

$\sin(2x + x) =$

$\sin 2x \cos x + \cos 2x \sin x =$

$2\sin x\cos x \cdot \cos x + (1 - 2\sin^2 x)\sin x =$

$2\sin x\cos^2 x + \sin x - 2\sin^3 x =$

$\sin x(2\cos^2 x + 1 - 2\sin^2 x) =$

$\sin x\,[2(1-\sin^2 x) + 1 - 2\sin^2 x] =$

$\sin x\,(2 - 2\sin^2 x + 1 - 2\sin^2 x) =$

$\sin x\,(3 - 4\sin^2 x) =$

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More Practice

Using Double Angle Identities

http://www.intmath.com/analytic-trigonometry/3-double-angle-formulas.php
http://www.ck12.org/trigonometry/Solving-Equations-with-Double-Angle-Identities/lesson/Solving-Trig-Equations-using-Double-and-Half-Angle-Formulas-ALG-II/
https://www.sophia.org/concepts/solving-an-equation-by-applying-a-double-angle-identity
https://www.youtube.com/watch?v=LSh_Ol_XsaE
https://www.youtube.com/watch?v=rF36a8K_3QM
https://www.youtube.com/watch?v=9mfvng-9cr0

Homework Assignment

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