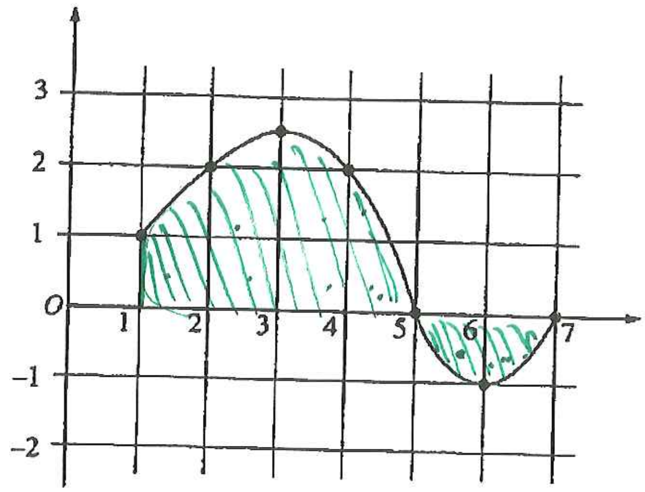
1995 AB6



The graph of a differentiable function f on the closed interval $[1,7]$ is shown above. Let $h(x)=\int_1^x f(t)\,dt$ for $1 \le x \le 7$.

(a) Find $h(1)$.

(b) Find $h'(4)$.

(c) On what interval or intervals is the graph of h concave upward? Justify your answer.

(d) Find the value of x at which h has its minimum on the closed interval $[1,7]$. Justify your answer.

a) $h(1)=\int_1^1 f(t)\,dt$

$\boxed{h(1)=0}$

b) $h'(x)=\frac{d}{dx}\int_1^x f(t)\,dt$

$h'(x)=f(x)$

$h'(4)=f(4)$

$\boxed{h'(4)=2}$

c) $h'(x)=f(x)$

$h''(x)=f'(x)>0 \rightarrow h$ conc. up

h concave up on $(1,3)\cup(6,7)$ b/c $h''(x)=f'(x)>0$ on $(1,3)\cup(6,7)$

d) $h'(x)=f(x)=0$

@ $x=5$, $x=7 \rightarrow$ crit #s

min value @ crit #s or endpts

$h(1)=0$ $\longrightarrow$ ∴, h has min value @ $x=1$

$h(5)=\int_1^5 f(x)\,dx>0$

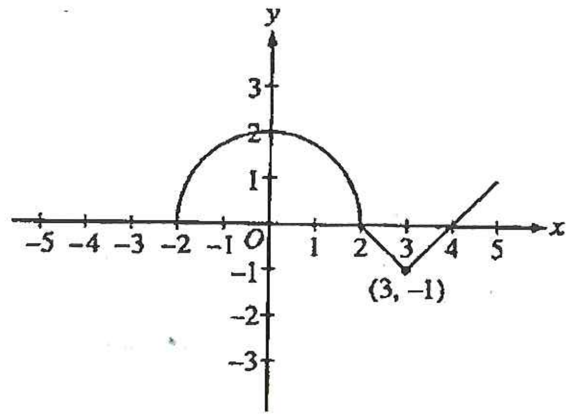
$h(7)=\int_1^7 f(x)\,dx$

$=\int_1^5 f(x)\,dx+\int_5^7 f(x)\,dx$

$\hookrightarrow \int_1^5 f(x)\,dx > \left|\int_5^7 f(x)\,dx\right|$

so, $h(7)>0$

1997 AB5/BC5



The graph of the function f consists of a semicircle and two line segments as shown above. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(3)$.

(b) Find all the values of x on the open interval $(-2, 5)$ at which g has a relative maximum. Justify your answer.

(c) Write an equation for the line tangent to the graph of g at $x = 3$.

(d) Find the x-coordinate of each point of inflection of the graph of g on the open interval $(-2, 5)$. Justify your answer.

a) $g(3) = \int_0^3 f(t)\,dt$

$= \frac{1}{4}\pi(2)^2 + \frac{1}{2}(1)(-1)$

$\boxed{g(3) = \pi - \frac{1}{2}}$

b) $g'(x) = \frac{d}{dx}\int_0^x f(t)\,dt$

$g'(x) = f(x) = 0$

@ $x = -2, 2, 4$ (crit #s)

$g' = f$: ? | -2 | + | 2 | − | 4 | +

g has rel. max @ $x = 2$ b/c $g'(x) = f(x)$ changes from pos. to neg @ $x = 2$

c) $y - y_1 = m(x - x_1)$

$y - g(3) = g'(3)(x - 3)$

$\boxed{y - (\pi - \frac{1}{2}) = -1(x - 3)}$

$g(3) = \int_0^3 f(t)\,dt$

$= \pi - \frac{1}{2}$

$g'(x) = f(x)$

$g'(3) = f(3)$

$= -1$

d) $g''(x) = f'(x) = 0$ or DNE

@ $x = -2, 0, 2, 3$ (p.i.p's)

$g'' = f'$: ? | -2 | + | 0 | − | 2 | − | 3 | +

g has inf. pt. @ $x = 0$ and $x = 3$ b/c $g''(x) = f'(x)$ changes signs @ $x = 0$ and $x = 3$