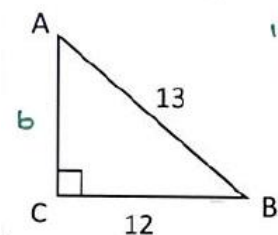


## 5.5 The Law of Sines

Target 6D: Use Law of Sines and Law of Cosines to solve triangles

Review Prior Concepts

Solve the triangle for all missing sides and angles.



$$12^2 + b^2 = 13^2$$

$$144 + b^2 = 169$$

$$b^2 = 25$$

$$b = 5$$

$$\sin A = \frac{12}{13}$$

$$A = \sin^{-1}\left(\frac{12}{13}\right)$$

$$\angle A = 67.38^\circ$$

$$\cos B = \frac{12}{13}$$

$$B = \cos^{-1}\left(\frac{12}{13}\right)$$

$$\angle B = 22.62^\circ$$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

With what given conditions can Law of Sines be used?

\* Angle-Side-Angle

\* Angle-Angle-Side

and Side-Side-Angle

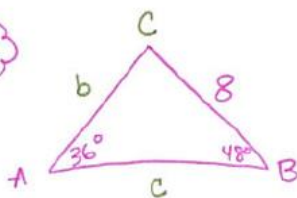
see back ... ☺

Example

Solve the triangle given  $\angle A = 36^\circ$ ,  $\angle B = 48^\circ$ ,  $a = 8$ .

☺ Draw picture

☺ AAS



$$\angle C = ?$$

$$b = ?$$

$$c = ?$$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle C = 180 - (\angle A + \angle B)$$

$$= 180 - (36 + 48)$$

$$\angle C = 96^\circ$$

$$\frac{\sin 36^\circ}{8} = \frac{\sin 48^\circ}{b}$$

$$b \sin 36^\circ = 8 \sin 48^\circ$$

$$b = \frac{8 \sin 48^\circ}{\sin 36^\circ}$$

$$b = 10.115$$

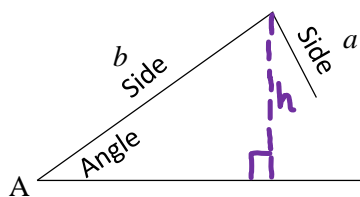
$$\frac{\sin 36^\circ}{8} = \frac{\sin 96^\circ}{c}$$

$$c \sin 36^\circ = 8 \sin 96^\circ$$

$$c = \frac{8 \sin 96^\circ}{\sin 36^\circ}$$

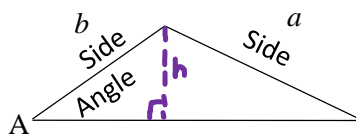
$$c = 13.536$$

The 3 Situations of Side-Side-Angle (The Ambiguous Case)



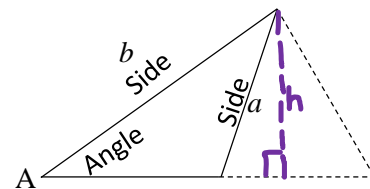
**No Triangle**

If  $a < h$ , where  $\sin A = \frac{h}{b}$ .



**One Triangle**

If  $a > h$ , where  $\sin A = \frac{h}{b}$ ,  
AND  $a \geq b$ .



**Two Triangles**

If  $a > h$ , where  $\sin A = \frac{h}{b}$ ,  
AND  $a < b$ .

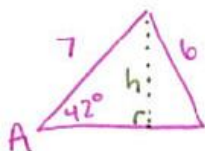
Examples

How many triangles can be made from the given information?

1.  $\angle A = 42^\circ, a = 6, b = 7$

2.  $\angle A = 42^\circ, a = 4, b = 5$

Draw picture  
SSA



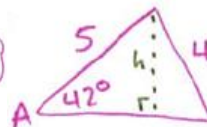
$$\sin 42^\circ = \frac{h}{7}$$

$$7 \sin 42^\circ = h$$

$$h = 4.684$$

$6 > 4.684$ ,  
but  $6 < 7$   $\therefore$  2  $\Delta$ s

Draw picture  
SSA



$$\sin 42^\circ = \frac{h}{5}$$

$$5 \sin 42^\circ = h$$

$$h = 3.346$$

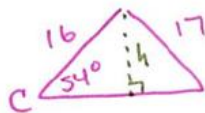
$4 > 3.346$ ,  
but  $4 < 5$   $\therefore$  2  $\Delta$ s

Compare a with h  
and a with b

3.  $\angle C = 54^\circ, b = 16, c = 17$

4.  $\angle C = 54^\circ, b = 4, c = 5$

Draw picture  
SSA



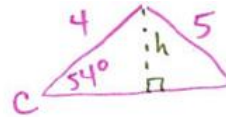
$$\sin 54^\circ = \frac{h}{16}$$

$$16 \sin 54^\circ = h$$

$$h = 12.944$$

$17 > 12.944$   
but  $17 \geq 16$   $\therefore$  one  $\Delta$

Draw picture  
SSA



$$\sin 54^\circ = \frac{h}{4}$$

$$4 \sin 54^\circ = h$$

$$h = 3.236$$

$5 > 3.236$   
but  $5 \geq 4$   $\therefore$  one  $\Delta$

Solve each triangle with the given information or state that a triangle cannot be made.

(there may be one  $\Delta$ , two  $\Delta$ s, or no  $\Delta$ )

5)  $\angle A = 30^\circ, a = 6, b = 7$

Draw picture

SSA

$$\sin 30^\circ = \frac{h}{7}$$

$$7 \sin 30^\circ = h$$

$$3.5 = h$$

$$\left. \begin{matrix} 6 > 3.5 \\ 6 < 7 \end{matrix} \right\} \therefore 2 \Delta s$$

Triangle 1

$\angle B = ?$   
 $\angle C = ?$   
 $c = ?$

$$\frac{\sin 30^\circ}{6} = \frac{\sin B}{7}$$

$$7 \sin 30^\circ = 6 \sin B$$

$$\frac{7 \sin 30^\circ}{6} = \sin B$$

$$\sin^{-1}\left(\frac{7 \sin 30^\circ}{6}\right) = B$$

$$\angle B = 35.685^\circ$$

$$\angle C = 180^\circ - (30^\circ + 35.685^\circ)$$

$$\angle C = 114.315^\circ$$

$$\frac{\sin 30^\circ}{6} = \frac{\sin 114.315^\circ}{c}$$

$$c \sin 30^\circ = 6 \sin 114.315^\circ$$

$$c = \frac{6 \sin 114.315^\circ}{\sin 30^\circ}$$

$$c = 10.936$$

Triangle 2

$\angle B' = 180^\circ - (35.685^\circ)$   
 $\angle B' = 144.315^\circ$   
 $\angle C' = 180^\circ - (30^\circ + 144.315^\circ)$   
 $\angle C' = 5.685^\circ$

$$\frac{\sin 30^\circ}{6} = \frac{\sin 5.685^\circ}{c}$$

$$c \sin 30^\circ = 6 \sin 5.685^\circ$$

$$c = \frac{6 \sin 5.685^\circ}{\sin 30^\circ}$$

$$c = 1.189$$

6)  $\angle B = 65^\circ, b = 11, c = 8$

Draw picture

SSA

$$\sin 65^\circ = \frac{h}{8}$$

$$8 \sin 65^\circ = h$$

$$h = 7.25$$

$$\left. \begin{matrix} 11 > 7.25 \\ 11 > 8 \end{matrix} \right\} \therefore \text{One } \Delta$$

$\angle C = ?$   
 $\angle A = ?$   
 $a = ?$

$$\frac{\sin 65^\circ}{11} = \frac{\sin C}{8}$$

$$8 \sin 65^\circ = 11 \sin C$$

$$\frac{8 \sin 65^\circ}{11} = \sin C$$

$$\sin^{-1}\left(\frac{8 \sin 65^\circ}{11}\right) = C$$

$$\angle C = 41.234^\circ$$

$$\angle B = 180^\circ - (65^\circ + 41.234^\circ)$$

$$\angle B = 73.766^\circ$$

$$\frac{\sin 65^\circ}{11} = \frac{\sin 73.766^\circ}{a}$$

$$a \sin 65^\circ = 11 \sin 73.766^\circ$$

$$a = \frac{11 \sin 73.766^\circ}{\sin 65^\circ}$$

$$a = 11.653$$

7)  $\angle C = 65^\circ, a = 7, c = 8$

Draw picture

SSA

$$\sin 65^\circ = \frac{h}{8}$$

$$h = 8 \sin 65^\circ$$

$$h = 6.344$$

$$\left. \begin{matrix} 8 > 6.344 \\ 8 > 7 \end{matrix} \right\} \therefore \text{one } \Delta$$

$\angle A = ?$   
 $\angle B = ?$   
 $b = ?$

$$\frac{\sin 65^\circ}{8} = \frac{\sin A}{7}$$

$$7 \sin 65^\circ = 8 \sin A$$

$$\frac{7 \sin 65^\circ}{8} = \sin A$$

$$\sin^{-1}\left(\frac{7 \sin 65^\circ}{8}\right) = A$$

$$\angle A = 52.469^\circ$$

$$\angle B = 180^\circ - (65^\circ + 52.469^\circ)$$

$$\angle B = 62.531^\circ$$

$$\frac{\sin 65^\circ}{8} = \frac{\sin 62.531^\circ}{b}$$

$$b \sin 65^\circ = 8 \sin 62.531^\circ$$

$$b = \frac{8 \sin 62.531^\circ}{\sin 65^\circ}$$

$$b = 7.832$$

**More Practice**

**Law of Sines**

<https://www.khanacademy.org/math/geometry/hs-geo-trig/hs-geo-law-of-sines/v/law-of-sines>

<https://www.mathsisfun.com/algebra/trig-sine-law.html>

<http://www.regentsprep.org/regents/math/algtrig/att12/lawofsines.htm>

<http://www.themathpage.com/atrig/law-of-sines.htm>

[http://www.softschools.com/math/calculus/the\\_ambiguous\\_case\\_of\\_the\\_law\\_of\\_sines/](http://www.softschools.com/math/calculus/the_ambiguous_case_of_the_law_of_sines/)

<https://www.youtube.com/watch?v=yVquId7xJQY>

<https://www.youtube.com/watch?v=ksBaHrVqhyo>

<https://www.youtube.com/watch?v=S4xAKewlqA4>

**Homework Assignment**

p.484 #1,3,7,11,13,15,19,25,29,33