

2008 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

| | | | | | | | |
|-----------------|-----|-----|-----|-----|-----|----|---|
| t (hours) | 0 | 1 | 3 | 4 | 7 | 8 | 9 |
| $L(t)$ (people) | 120 | 156 | 176 | 126 | 150 | 80 | 0 |

2. Concert tickets went on sale at noon ($t = 0$) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \leq t \leq 9$. Values of $L(t)$ at various times t are shown in the table above.

- (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ($t = 5.5$). Show the computations that lead to your answer. Indicate units of measure.
- (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

$$a) L'(5.5) = \frac{L(7) - L(4)}{7 - 4} = \frac{150 - 126}{3} = 8 \text{ people/hr}$$

$$b) \text{Avg}^{\#} \text{ of people} = \frac{1}{b-a} \int_a^b f(x) dx = \frac{1}{4-0} \int_0^4 L(t) dt$$

$$= \frac{1}{4} \left[\frac{1}{2}(156+120)(1) + \frac{1}{2}(176+156)(2) + \frac{1}{2}(126+176)(1) \right]$$

$$= \boxed{155.25 \text{ people}}$$

2008 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

| | | | | | |
|---------------------------------------|---|---|----|----|----|
| Distance from the river's edge (feet) | 0 | 8 | 14 | 22 | 24 |
| Depth of the water (feet) | 0 | 7 | 8 | 2 | 0 |

3. A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin(\sqrt{t} + 10)$ for $0 \leq t \leq 120$ minutes.

(a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

(b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

$$a) \text{trapez. area of cross section of River} = \frac{1}{2}(7+0)(8) + \frac{1}{2}(8+7)(6) + \frac{1}{2}(2+8)(6) + \frac{1}{2}(0+2)(2)$$

$$= 115 \text{ ft}^2$$

$$b) \text{Avg value} = \frac{1}{120-0} \int_0^{120} 115 v(t) dt$$

$$= \frac{1}{120} \int_0^{120} 115 v(t) dt = \boxed{1807.169 \text{ ft}^3/\text{min}}$$