PCM 30S Unit 1 ~~Name~~ Answer Key

Law of Sines - Ambiguous Case

Date_____

Find each measurement indicated. Round your answers to the nearest tenth.

1) $m\angle A = 28°, c = 34, a = 32$
Find b

SSA

$\sin 28 = \frac{h}{34}$
$h = 34\sin 28°$
$h = 15.962$

$32 > 15.962$
$32 < 34$
$\therefore$ 2 Δs

Triangle 1

$\frac{\sin 28°}{32} = \frac{\sin C}{34}$
$34\sin 28° = 32\sin C$
$\frac{34\sin 28°}{32} = \sin C$
$\angle C = \sin^{-1}\left(\frac{34\sin 28°}{32}\right)$
$\angle C = 29.922°$
$\angle B = 180 - (28° + 29.922°)$
$\angle B = 128.078°$
$\frac{\sin 28°}{32} = \frac{\sin 128.078°}{b}$
$b\sin 28° = 32\sin 128.078°$
$b = \frac{32\sin 128.078°}{\sin 28°}$
$b = 57.755$

Triangle 2

$\angle C' = 180 - 29.922°$
$\angle C' = 150.078°$
$\angle B' = 180 - (28° + 150.078°)$
$\angle B' = 1.922°$
$\frac{\sin 28°}{32} = \frac{\sin 1.922°}{b}$
$b\sin 28° = 32\sin 1.922°$
$b = \frac{32\sin 1.922°}{\sin 28°}$
OR
$b = 2.286$

2) $m\angle B = 23°, a = 30, b = 25$
Find c

SSA

$\sin 23° = \frac{h}{30}$
$30\sin 23° = h$
$h = 11.722$

$25 > 11.722$
and $25 < 30$
$\therefore$ 2 Δs

Triangle 1

$\frac{\sin 23°}{25} = \frac{\sin A}{30}$
$30\sin 23° = 25\sin A$
$\frac{30\sin 23°}{25} = \sin A$
$\angle A = \sin^{-1}\left(\frac{30\sin 23°}{25}\right)$
$\angle A = 27.961°$
$\angle C = 180 - (23 + 27.961°)$
$\angle C = 129.039°$
$\frac{\sin 23°}{25} = \frac{\sin 129.039°}{c}$
$c\sin 23° = 25\sin 129.039°$
$c = \frac{25\sin 129.039°}{\sin 23°}$
$c = 49.697$

Triangle 2

$\angle A' = 180 - 27.961°$
$\angle A' = 152.039°$
$\angle C' = 180 - (23 + 152.039)$
$\angle C' = 4.961°$
$\frac{\sin 23°}{25} = \frac{\sin 4.961°}{c'}$
$c'\sin 23° = 25\sin 4.961°$
$c' = \frac{25\sin 4.961°}{\sin 23°}$
OR
$c' = 5.534$

3) $m\angle C = 9°, m\angle A = 151°, b = 24$
Find a

ASA

one Δ.

$\angle B = 180° - (9° + 151°)$
$\angle B = 20°$
$\frac{\sin 20°}{24} = \frac{\sin 151°}{a}$
$a\sin 20° = 24\sin 151°$
$a = \frac{24\sin 151°}{\sin 20°}$
$a = 34.020$

4) $m\angle C = 48°, b = 32, c = 31$
Find a

SSA

$\sin 48° = \frac{h}{32}$
$32\sin 48° = h$
$h = 23.781$

$31 > 23.781$
and $31 < 32$
$\therefore$ 2 Δs

Triangle 1

$\frac{\sin 48°}{31} = \frac{\sin B}{32}$
$32\sin 48° = 31\sin B$
$\frac{32\sin 48°}{31} = \sin B$
$\angle B = \sin^{-1}\left(\frac{32\sin 48°}{31}\right)$
$\angle B = 50.096°$
$\angle C = 180 - (48° + 50.096°)$
$\angle C = 81.904°$
$\frac{\sin 48°}{31} = \frac{\sin 81.904°}{c}$
$c\sin 48° = 31\sin 81.904°$
$c = \frac{31\sin 81.904°}{\sin 48°}$
$c = 41.299$

Triangle 2

$\angle B' = 180 - 50.096°$
$\angle B' = 129.904°$
$\angle C' = 180 - (48° + 129.904°)$
$\angle C' = 2.096°$
$\frac{\sin 48°}{31} = \frac{\sin 2.096°}{c'}$
OR
$c' = \frac{31\sin 2.096}{\sin 48°}$
$c' = 1.525$

5) $m\angle C = 17°, m\angle A = 15°, c = 27$
Find a

AAS

one Δ

$\frac{\sin 17°}{27} = \frac{\sin 15°}{a}$
$a\sin 17° = 27\sin 15°$
$a = \frac{27\sin 15°}{\sin 17°}$
$a = 23.901$

6) $m\angle A = 41°, c = 32, a = 31$
Find b

SSA

$\sin 41° = \frac{h}{32}$
$h = 32\sin 41°$
$h = 20.994$

$31 > 20.994$
and $31 < 32$
$\therefore$ 2 Δs

Triangle 1

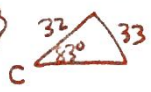
$\frac{\sin 41°}{31} = \frac{\sin C}{32}$
$32\sin 41° = 31\sin C$
$\frac{32\sin 41°}{31} = \sin C$
$\angle C = \sin^{-1}\left(\frac{32\sin 41°}{31}\right)$
$\angle C = 42.627°$
$\angle B = 180 - (41° + 42.627°)$
$\angle B = 96.373°$
$\frac{\sin 41°}{31} = \frac{\sin 96.373°}{b}$
$b\sin 41° = 31\sin 96.373°$
$b = \frac{31\sin 96.373°}{\sin 41°}$
$b = 46.960$

Triangle 2

$\angle C' = 180 - 42.627°$
$\angle C' = 137.373°$
$\angle B' = 180 - (41° + 137.373°)$
$\angle B' = 1.627°$
$\frac{\sin 41°}{31} = \frac{\sin 1.627°}{b'}$
$b'\sin 41° = 31\sin 1.627°$
$b' = \frac{31\sin 1.627°}{\sin 41°}$
OR
$b' = 1.342$

7) $m\angle C = 83°$, $b = 32$, $c = 33$
Find $m\angle B$

SSA



$\sin 83° = \frac{h}{32}$

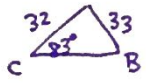
$32\sin 83° = h$

$h = 31.768$

$33 > 31.761$

$33 > 32$

$\therefore$ one Δ



$\frac{\sin 83°}{33} = \frac{\sin B}{32}$

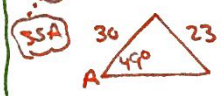
$33\sin B = 32\sin 83°$

$\sin B = \frac{32\sin 83}{33}$

$\angle B = \sin^{-1}\left(\frac{32\sin 83}{33}\right) \Rightarrow$ **$\angle B = 74.253°$**

8) $m\angle A = 49°$, $c = 30$, $a = 23$
Find $m\angle B$

SSA



$\sin 49° = \frac{h}{30}$

$h = 30\sin 49°$

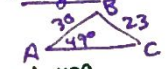
$h = 22.641$

$23 > 22.641$

$23 < 30$

$\therefore$ 2 Δs

Triangle 1



$\frac{\sin 49°}{23} = \frac{\sin C}{30}$

$30\sin 49° = 23\sin C$

$\frac{30\sin 49°}{23} = \sin C$

$\angle C = \sin^{-1}\left(\frac{30\sin 49°}{23}\right)$

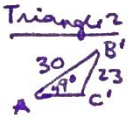
$\angle C = 79.868°$

$\angle B = 180 - (49 + 79.868°)$

$\angle B = 51.132°$

OR

Triangle 2



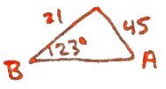
$\angle C' = 180 - 79.868°$

$\angle C' = 100.132°$

$\angle B' = 180 - (49° + 100.132°)$

$\angle B' = 30.868°$

9) $m\angle B = 123°$, $a = 21$, $b = 45$
Find $m\angle A$



$\sin 123° = \frac{h}{21}$

$h = 21\sin 123°$

$h = 17.612$

$45 > 17.612$

$45 > 21$

$\therefore$ one Δ

$\frac{\sin 123°}{45} = \frac{\sin A}{21}$

$21\sin 123° = 45\sin A$

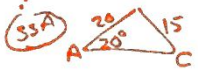
$\frac{21\sin 123°}{45} = \sin A$

$\angle A = \sin^{-1}\left(\frac{21\sin 123°}{45}\right)$

$\angle A = 23.040°$

10) $m\angle A = 20°$, $c = 20$, $a = 15$
Find $m\angle B$

SSA



$\sin 20° = \frac{h}{20}$

$h = 20\sin 20°$

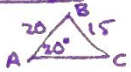
$h = 6.840$

$15 > 6.840$

$15 < 20$

$\therefore$ 2 Δs

Triangle 1



$\frac{\sin 20°}{15} = \frac{\sin C}{20}$

$15\sin C = 20\sin 20°$

$\sin C = \frac{20\sin 20°}{15}$

$\angle C = \sin^{-1}\left(\frac{20\sin 20°}{15}\right)$

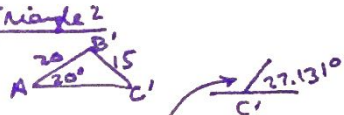
$\angle C = 27.131°$

$\angle B = 180 - (20° + 27.131°)$

$\angle B = 132.869°$

OR

Triangle 2



$\angle C' = 180 - 27.131°$

$\angle C' = 152.869°$

$\angle B' = 180 - (20 + 152.869°)$

$\angle B' = 7.131°$

11) $m\angle C = 51°$, $b = 33$, $c = 28$
Find $m\angle A$

SSA

$\sin 51° = \frac{h}{33}$

$h = 33\sin 51°$

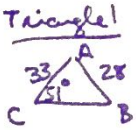
$h = 25.646$

$28 > 25.646$

$28 < 33$

$\therefore$ 2 Δs

Triangle 1



$\frac{\sin 51°}{28} = \frac{\sin B}{33}$

$33\sin 51° = 28\sin B$

$\frac{33\sin 51°}{28} = \sin B$

$\angle B = \sin^{-1}\left(\frac{33\sin 51°}{28}\right)$

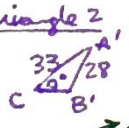
$\angle B = 66.337°$

$\angle A = 180 - (51 + 66.337°)$

$\angle A = 62.663°$

OR

Triangle 2



$\angle B' = 180 - 66.337°$

$\angle B' = 113.663°$

$\angle A' = 180 - (51° + 113.663°)$

$\angle A' = 15.337°$

12) $m\angle C = 109°$, $b = 14$, $c = 22$
Find $m\angle B$

SSA

$\sin 109° = \frac{h}{14}$

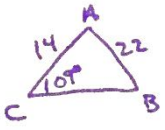
$h = 14\sin 109°$

$h = 13.237$

$22 > 13.237$

$22 > 14$

$\therefore$ one Δ



$\frac{\sin 109°}{22} = \frac{\sin B}{14}$

$14\sin 109° = 22\sin B$

$\frac{14\sin 109°}{22} = \sin B$

$\angle B = \sin^{-1}\left(\frac{14\sin 109°}{22}\right)$

$\angle B = 36.991°$