DATE: _____

AP M/C & FRQ Trapezoidal Approximation Questions

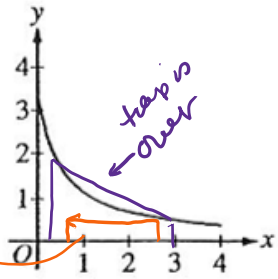
1.

If a trapezoidal sum overapproximates $\int_0^4 f(x)\,dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$?

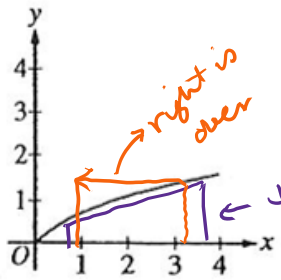
f(x) is concave up

f(x) is dec

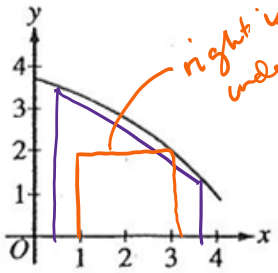
(A)



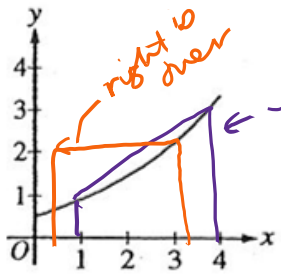
(B)



(C)



(D)



(E)

choice A b/c when f(x) is concave up trap sum overapprox and when f(x) is dec right sum underapprox the $\int_0^4 f(x)\,dx$.

2.

x	$f(x)$	$f'(x)$
0	49	0
1	2	–8
2	–1	–80

f' dec → f''<0 → f concave down

The table above gives selected values for a differentiable and decreasing function f and its derivative. Which of the following is true?

I. A trapezoidal sum overapproximates $\int_0^2 f(x)\,dx$ → False b/c trapezoid sum underapprox $\int_0^2 f(x)dx$ when f'(x) dec

II. A left Riemann sum underapproximates $\int_0^2 f(x)dx$ → False b/c left sum overapprox of $\int_0^2 f(x)dx$ when f(x) is dec

(A) I only **(B)** II only **(C)** I and II are both true **(D)** Neither I nor II are true

3.

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

avg value $\frac{1}{b-a}\int_a^b f(x)\,dx$

$$\text{avg temp of wire} = \frac{1}{8-0}\int_0^8 T(x)\,dx$$

$$\approx \frac{1}{8}\left(\frac{1}{2}(100+93)(1)+\frac{1}{2}(93+70)(4)+\frac{1}{2}(70+62)(1)+\frac{1}{2}(62+55)(2)\right)$$

$$\approx \frac{1}{8}\left(\frac{1}{2}(193+163(4)+132+117(2)\right)$$

$$\approx \frac{1}{8}\left(\frac{1}{2}(193+652+132+234)\right)$$

$$\approx \frac{1}{8}\left(\frac{1}{2}(1211)\right)$$

$$\approx \boxed{\frac{1211}{16}\ ^\circ C} \text{ or } \boxed{75.688\ ^\circ C}$$