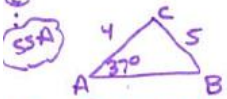
5.6 The Law of Cosines

Target 6D: Use Law of Sines and Law of Cosines to solve triangles

Review Prior Concepts

1. Given ∠A= 37°, a = 5, b = 4, find the value of ∠C.



$\sin 37° = \frac{h}{4}$

$h = 4\sin 37°$

$h = 2.407$

$5 > 2.407$

$5 > 4$

∴, one △.

$\frac{\sin 37°}{5} = \frac{\sin B}{4}$

$4\sin 37° = 5\sin B$

$\frac{4\sin 37°}{5} = \sin B$

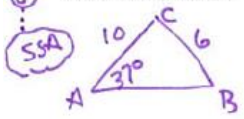
$\angle B = \sin^{-1}\left(\frac{4\sin 37°}{5}\right)$

$\angle B = 28.780°$

$\angle C = 180 - (37 + 28.780°)$

$\angle C = 114.220°$

2. Given ∠A= 37°, a = 6, b = 10, find the value of ∠C.



$\sin 37° = \frac{h}{10}$

$h = 10\sin 37°$

$h = 6.018$

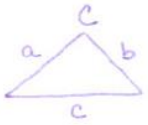
$6 < 6.018$

∴, no △

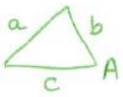
∠C DNE

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$



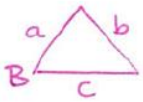
OR



$a^2 = b^2 + c^2 - 2bc\cos A$

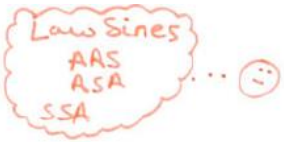
OR

$b^2 = a^2 + c^2 - 2ac\cos B$



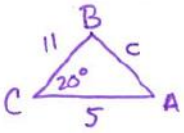
➲With what given conditions can Law of Cosines be used?

* Side – Side – Side (SSS)
* Side – Angle – Side (SAS)



Examples

1) Solve the triangle given ∠C = 20°, a = 11cm, and b = 5cm.



$c = ?$

$\angle A = ?$

$\angle B = ?$

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 11^2 + 5^2 - 2(11)(5)\cos 20°$

$c = \sqrt{11^2 + 5^2 - 2(11)(5)\cos 20°}$

$c = 6.529$ cm

"sto"

$a^2 = b^2 + c^2 - 2bc\cos A$

$11^2 = 5^2 + (6.529)^2 - 2(5)(6.529)\cos A$

$11^2 - 5^2 - (6.529)^2 = -2(5)(6.529)\cos A$

$\frac{11^2 - 5^2 - (6.529)^2}{-2(5)(6.529)} = \cos A$

$\angle A = \cos^{-1}\left(\frac{11^2 - 5^2 - (6.529)^2}{-2(5)(6.529)}\right)$

$\angle A = 144.817°$

$\angle B = 180 - (20° + 144.817°)$

$\angle B = 15.183°$

2) Solve the triangle given $a = 7$, $b = 8$, and $c = 10$.

$c^2 = a^2 + b^2 - 2ab\cos C$

$10^2 = 7^2 + 8^2 - 2(7)(8)\cos C$

$100 = 113 - 112\cos C$

$-13 = -112\cos C$

$\frac{13}{112} = \cos C$

$\angle C = \cos^{-1}\left(\frac{13}{112}\right)$

$\angle C = 83.335°$

$a^2 = b^2 + c^2 - 2bc\cos A$

$7^2 = 8^2 + 10^2 - 2(8)(10)\cos A$

$7^2 - 8^2 - 10^2 = -2(8)(10)\cos A$

$\frac{7^2 - 8^2 - 10^2}{-2(8)(10)} = \cos A$

$\angle A = \cos^{-1}\left(\frac{7^2 - 8^2 - 10^2}{-2(8)(10)}\right)$

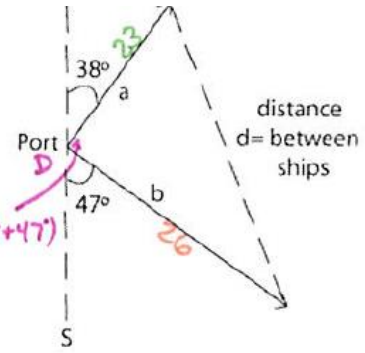
$\angle A = 44.049°$

$\angle B = 180 - (83.335° + 44.049°)$

$\angle B = 52.617°$

Applications

Two ships leave port at 4 p.m. One is headed at a bearing of N 38 E and is traveling at 11.5 miles per hour. The other is traveling 13 miles per hour at a bearing of S 47 E. How far apart are they when dinner is served at 6 p.m.?



time = 2 hrs

$a = \left(11.5\frac{\text{miles}}{\text{hr}}\right)(2\text{hr}) = 23 \text{ miles}$

$b = \left(13\frac{\text{miles}}{\text{hr}}\right)(2\text{hr}) = 26 \text{ miles}$

$\angle D = 180 - (38 + 47)$

$\angle D = 95°$

$d^2 = a^2 + b^2 - 2ab\cos D$

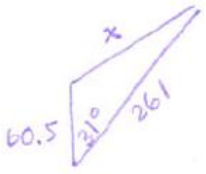
$d^2 = 23^2 + 26^2 - 2(23)(26)\cos 95°$

$d = \sqrt{23^2 + 26^2 - 2(23)(26)\cos 95°}$

$d = 36.183$

The two ships are 36.183 miles apart when dinner is served

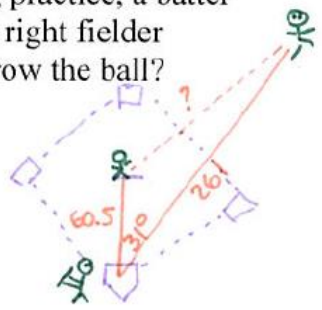
2) On a baseball field, the pitcher's mound is 60.5 feet from home plate. During practice, a batter hits a ball 261 feet at an angle of 31° to the right of the pitcher's mound. The right fielder catches the ball and throws it to the pitcher. How far does the right fielder throw the ball?



$x^2 = 60.5^2 + 261^2 - 2(60.5)(261)\cos 31°$

$x = \sqrt{60.5^2 + 261^2 - 2(60.5)(261)\cos 31°}$

$x = 211.45$



The right fielder throws the ball 211.45 ft

More Practice

Law of Cosines

https://www.mathsisfun.com/algebra/trig-cosine-law.html
http://www.mathwarehouse.com/trigonometry/law-of-cosines-formula-examples.php
http://www.regentsprep.org/regents/math/algtrig/att12/lawofcosines.htm
https://www.khanacademy.org/math/geometry/hs-geo-trig/hs-geo-law-of-cosines/e/law_of_cosines
https://www.youtube.com/watch?v=ZElOxG7_m3c
https://www.youtube.com/watch?v=ZElOxG7_m3c
https://www.youtube.com/watch?v=QkpDJaze31k

Homework Assignment

p.494 #1,3,5,9-17 odd, 21