DATE: ____________________

## Solutions to Differential Equations

Find the general solution to the differential equation.

**(1)** Solve for the differential ($dy$).

**(2)** Anti-derive both sides of the equation.

*Example 1:*

Find the general solution to $\frac{dy}{dx} = 2x$.

$\int dy = \int 2x\, dx$

$\boxed{y = x^2 + C}$

*Example 2:*

Find the general solution to $\frac{dy}{dx} = \cos x\, e^{\sin x}$.

$\int dy = \int \cos x\, e^{\sin x}\, dx$

$u = \sin x$

$du = \cos x\, dx$

$y = \int e^u\, du$

$y = e^u + C$

$\boxed{y = e^{\sin x} + C}$

*Example 3:*

Find the general solution to $\frac{dy}{dt} = \frac{1}{1-2t}$.

$\int dy = \int \frac{1}{1-2t}\, dt$

$u = 1-2t$

$du = -2\, dt$

$-\frac{1}{2}\, du = dt$

$y = \int \frac{1}{u} \cdot -\frac{1}{2}\, du$

$y = -\frac{1}{2} \int \frac{1}{u}\, du$

$y = -\frac{1}{2} \ln|u| + C$

$\boxed{y = -\frac{1}{2} \ln|1-2t| + C}$ or $\boxed{y = \ln|1-2t|^{-1/2} + C}$

## Particular Solutions to Differential Equations

Find the particular solution to the differential equation (*or* Solve the initial value problem explicitly)

**(1)** Solve for the differential ($dy$).

**(2)** Anti-derive both sides of the equation.

**(3)** Sub in given values to solve for C.

**(4)** Write solution with particular value of C.

*Example 1:*

Find the particular solution to $\frac{dy}{dx} = 2x$ at $(1,3)$.

x y

$\int dy = \int 2x\, dx$

general solution → $y = x^2 + C$ → $\boxed{y = x^2 + 2}$ particular solution

$3 = 1^2 + C$

$3 = 1 + C$

$2 = C$

*Example 2:*

Solve the differential equation $\frac{dy}{dx} = \cos x - 3x^2$ with the initial value of $y = 3$ when $x = 0$.

$\int dy = \int (\cos x - 3x^2)\, dx$

general solution → $y = \sin x - x^3 + C$ → $\boxed{y = \sin x - x^3 + 3}$ particular solution

$3 = \sin 0 - 0^3 + C$

$3 = C$

*Example 3:*

Find the solution to $\frac{dA}{dt} = \cos t \sin t$ for $A = \frac{3}{2}$ for $t = \frac{\pi}{2}$.

$\int dA = \int \cos t \sin t\, dt$  $u = \sin t$, $du = \cos t\, dt$

$A = \int u\, du$

$A = \frac{1}{2}u^2 + C$

general solution → $A = \frac{1}{2}(\sin t)^2 + C$ → $\boxed{A = \frac{1}{2}\sin^2 t + 1}$ particular solution

$\frac{3}{2} = \frac{1}{2}(\sin \frac{\pi}{2})^2 + C$

$\frac{3}{2} = \frac{1}{2}(1)^2 + C$ → $\frac{3}{2} = \frac{1}{2} + C$

$1 = C$